ESoA course on

Large Scale Radio Propagation

General Theory of Propagation

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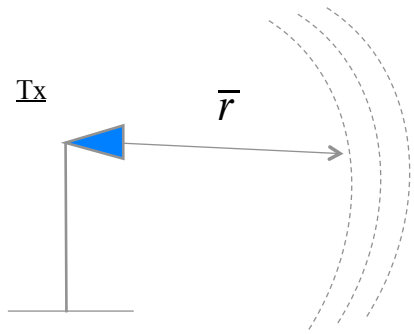
Radiation in free space

Far field: (r >> λ r >>D, antenna size)



(Spherical wave)

$$\vec{E}(r,\theta,\varphi) \approx \vec{E}_o(\theta,\varphi)\frac{e^{-j\beta r}}{r}$$

vectorial amplitude

$$\bar{E}_o \perp \bar{r}$$

propagation factor

$$\beta = \frac{2\pi}{\lambda}$$

$\vec{E}$ Complex vector or *phasor*, so that

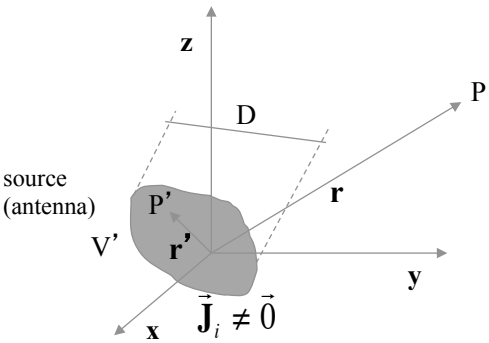
$$\vec{e}(r,\theta,\varphi,t) = \Re e\left\{\vec{E}(r,\theta,\varphi)e^{j\omega t}\right\}$$

Detailed far field expressions

$$\vec{E}(r,\theta,\varphi) \approx \underbrace{-j\frac{\eta}{2\lambda}\overbrace{\left[\ \hat{\mathrm{i}}_{\mathrm{r}} \times \vec{M}(\theta,\varphi) \times \hat{\mathrm{i}}_{\mathrm{r}}\right]}^{\vec{M}^t}}_{\vec{E}_o(\theta,\varphi)} \frac{e^{-j\beta r}}{r}$$

$$\vec{H}(r,\theta,\varphi) \approx \frac{1}{\eta}\hat{\mathrm{i}}_{\mathrm{r}} \times \vec{E} \qquad \vec{\mathrm{S}} = \frac{1}{2}\vec{E} \times \vec{H}^*$$

Poynting vector



Antenna's *radiation vector*

$$\vec{\mathrm{M}}(\theta,\varphi) \approx \int_{V'} \vec{\mathbf{J}}_i(P')\exp\left[j\beta\left(\vec{r}\,'\cdot\hat{\mathbf{i}}_r\right)\right]dV'$$

Polarization vector

$$\hat{\mathbf{p}} \triangleq \frac{\vec{E}}{\left|\vec{E}\right|} \cdot e^{j\chi}$$

The polarization vector defines the polarization of the field (and also of the antenna)

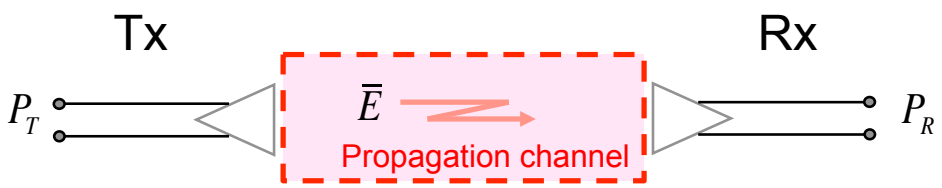
It is strictly related to the radiation vector **M**

$$\hat{\mathbf{p}} = \frac{\vec{E}}{\left|\vec{E}\right|} \cdot e^{j\chi} = \frac{-j\dfrac{\eta}{2\lambda r}\vec{M}^t\dfrac{e^{-j\beta r}}{r}}{\dfrac{\eta}{2\lambda r}\left|\vec{M}^t\right|\dfrac{1}{r}} \cdot e^{j\chi} = \frac{\vec{M}^t}{\left|\vec{M}^t\right|}e^{-j\left(\beta r+\pi/2\right)} \cdot e^{j\chi} \Rightarrow \hat{\mathbf{p}} = \frac{\vec{M}^t}{\left|\vec{M}^t\right|} = \hat{M}^t$$

Emitted field

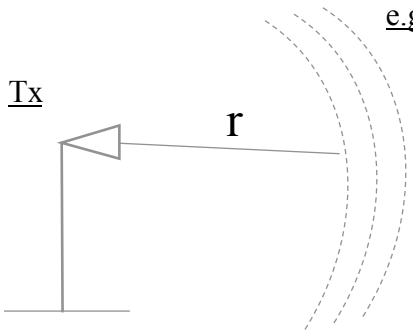
The field emitted by an antenna can be derived from the transmitted power P_T and the polarization vector using the following formula

$$\vec{E} = \sqrt{\frac{\eta P_T g_T}{2\pi}} \frac{e^{-j\beta r}}{r} \hat{\mathbf{p}} = \sqrt{60 P_T g_T} \frac{e^{-j\beta r}}{r} \hat{\mathbf{p}}$$



Wavefront

A *wavefront* is a *locus* where the field has constant phase



e.g:

$$E_x(r,\theta,\varphi) = E_{0x}(\theta,\varphi)\frac{e^{-j\beta r}}{r} = \frac{|E_{0x}|}{r} e^{j(\arg(E_{0x})-\beta r)}$$

Therefore the wavefront is defined by:

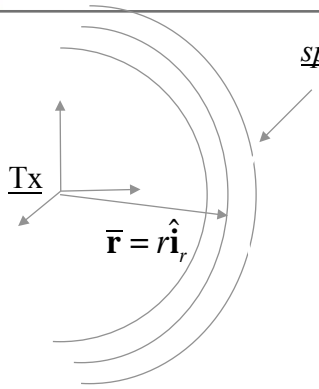
$$\arg(E_{0x}) - \beta r = -\bar{\varphi}$$

If r is large and $\arg(E_{0x})$ is small we have

$$\arg(E_{0x}) - \beta r \approx -\beta r = -\bar{\varphi} \;\Rightarrow\; r = \bar{\varphi}/\beta$$

Therefore the wavefront is a spherical surface. That's why it's called "spherical wave".

Spherical and plane waves

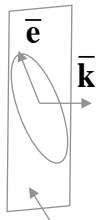


Spherical wave

$$\vec{E}(r,\theta,\varphi) \approx \vec{E}_o(\theta,\varphi)\frac{e^{-j\beta r}}{r}$$

In far field a spherical wave can be locally approximated with a Plane wave

$$\vec{E}(r,\theta,\varphi) \approx \vec{E}_o^{'} e^{-j\,\bar{\mathbf{k}}\cdot\bar{\mathbf{r}}}$$



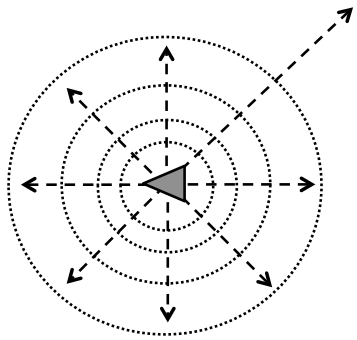
with

$$\vec{E}_o^{'} = \frac{\vec{E}_o(\theta,\varphi)}{r} \qquad \bar{\mathbf{k}} = \beta\hat{\mathbf{i}}_r$$

≈ plane wavefront

Definition of Ray (1/2)

- Given a propagating wave, every curve that is everywhere perpendicular to the wavefront is called ***electromagnetic ray***. A ray is *the path* of the wavefront. There is a mutual identification btw wave and ray
- Therefore we assume that the ray also "has" a field or "carries" a field, the field of the corresponding wave at every point

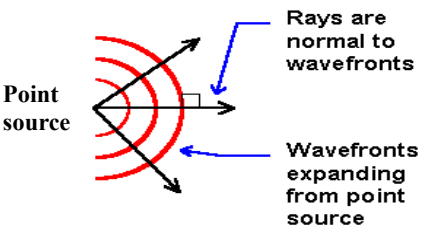


e.g: spherical wave and rays generated by a Tx antenna

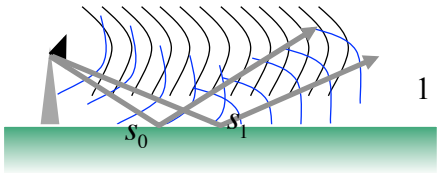
Definition of Ray (2/2)

- In free space, rays are rectilinear
- In presence of ***concentrated obstacles*** rays are piecewise-rectilinear and wavefronts can be of various kinds (see further on)
- In non-homogeneous media rays can be curved (not treated here)

Ex.1 Sperical wave and ***rectilinear*** *rays*

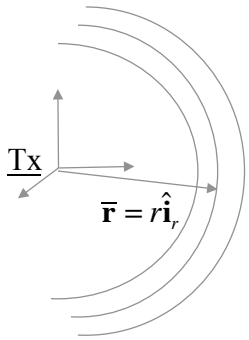


Ex.2 reflected spherical wave and ***piece-wise rectilinear*** *rays*



Poynting vector and power-density

The Poynting vector always has the same direction as the ray.
For a spherical wave it is easy to show:



$$\bar{S} = \frac{\bar{E} \times \bar{H}^*}{2} = \frac{1}{2}\bar{E} \times \left(\frac{1}{\eta}\hat{i}_r \times \bar{E}^* \right) = \frac{|\bar{E}|^2}{2\eta}\hat{i}_r$$

where we used the identity:

$$\bar{A} \times \bar{B} \times \bar{C} = \left(\bar{A} \cdot \bar{C}\right) \cdot \bar{B} - \left(\bar{A} \cdot \bar{B}\right) \cdot \bar{C}$$

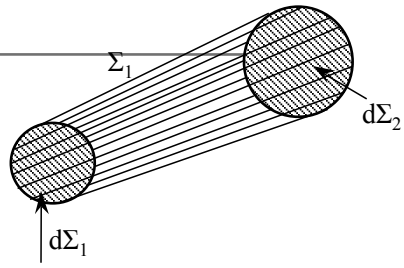
Therefore:
e.m. power propagates along rays!

radiation *Power-Density* or *Intensity* is defined as:

$$|\bar{S}| = S = \frac{|\bar{E}|^2}{2\eta}$$

Ray tube and power-density law

Def: a Ray tube (tube of flux) is a closed tube whose lateral surface is formed by a set of rays and the bases by two wavefront sections



By applying the Poynting's theorem (power balance) to a ray tube having bases $d\Sigma_1$, $d\Sigma_2$ small enough to assume the field constant on them, and considering a lossless medium we have:

$$\oint_{\Sigma} \vec{S}\cdot\hat{n}\ d\Sigma = \int_{d\Sigma_1} \vec{S}\cdot\hat{n}\ d\Sigma + \underbrace{\int_{\Sigma_\ell} \vec{S}\cdot\hat{n}\ d\Sigma}_{=0} + \int_{d\Sigma_2} \vec{S}\cdot\hat{n}\ d\Sigma =$$

$$= -\left|\vec{S}_1\right|\cdot d\Sigma_1 + \left|\vec{S}_2\right|\cdot d\Sigma_2 = 0 \quad \Rightarrow \quad S_1\cdot d\Sigma_1 = S_2\cdot d\Sigma_2 \Rightarrow \quad \frac{S_2}{S_1} = \frac{d\Sigma_1}{d\Sigma_2}$$

<u>Power-density (or Intensity) law of Geometrical Optics:</u>
"Power-density is inversely proportional to the cross-section of the ray tube"

Spreading Factor

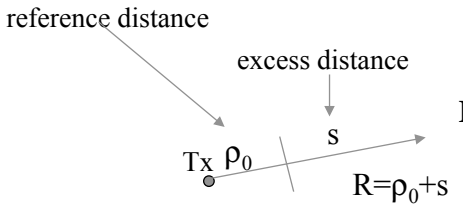
We can therefore define the *<u>spreading</u>* or *<u>divergence factor A:</u>*

$$A = \sqrt{\frac{\left|\bar{S}_2\right|}{\left|\bar{S}_1\right|}} = \frac{\left|\bar{E}_2\right|}{\left|\bar{E}_1\right|} = \sqrt{\frac{d\Sigma_1}{d\Sigma_2}}$$

The spreading factor accounts for the field attenuation due to the enlarging of the ray- tube cross-section.
The field carried by a ray attenuates even in lossless media because the power spreads over an enlarging wavefront surface as the wave propagates.

Re-writing the spherical wave

Using a reference distance ρ_o we have



field:

$$\mathbf{E}(R)=\mathbf{E}_0\frac{e^{-j\beta R}}{R}=\mathbf{E}_0\frac{e^{-j\beta\rho_0}}{\rho_0}\;\frac{\rho_0}{R}e^{-j\beta(R-\rho_0)}=$$

$$\mathbf{E}(\rho_0)\frac{\rho_0}{R}e^{-j\beta(R-\rho_0)}=\underbrace{\mathbf{E}(\rho_0)}_{\substack{\textit{Field}\text{ at reference}\\ \text{point (s=0)}}}\cdot\underbrace{\underbrace{\frac{\rho_0}{\rho_0+s}}_{\text{"}\textbf{Divergence factor}\text{"}}\cdot\underbrace{e^{-j\beta s}}_{\textit{Phase}\text{ factor}}}_{\textit{Propagation factor}}$$

Power density:

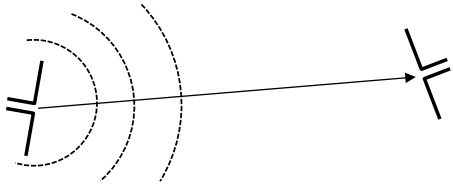
$$S(R)=S(\rho_0)\left(\frac{\rho_0}{\rho_0+s}\right)^2$$

Spreading factor

Received power P_r is proportional to power density, that's why P_r attenuates with the square of distance !

$$P_R=S\cdot A_{eff}\,;\qquad S\triangleq\frac{|\bar{E}|^2}{2\eta}$$

Free space radio link



- The simplest example of wireless channel is represented by the "Free Space" (FS) case; the Tx and the Rx are completely surrounded by an homogeneous, isotropic, linear and loss-less medium. Antennas are in **Line of Sight (LoS)** and propagation can be limited to the only *direct ray;*
- The radio link can be fully described by the **Friis** (or **Free Space**) **formula**:

$$\boxed{P_R=P_A\,g_T(\theta_T,\phi_T)\,g_R(\theta_R,\phi_R)\left(\frac{\lambda}{4\pi R}\right)^2\rho_D\;\tau_P}$$

"inverse power law"

- R: link distance
- P_A: power absorbed (radiated) by the transmitting antenna;
- g_T, g_R: antenna gains;
- λ: wavelength
- ρ_D: power matching factor;
- τ_P: polarization matching factor

Path Loss and Path Gain

- **Path Gain (PG)** : describes how the received power (field strength) decreases with link distance ➔ PG = PG(R);
- **Path Loss (PL)**: is the reciprocal of PG, i.e. it describes how the attenuation of the propagating signal increases with link distance ➔ PL = PL(R) = 1/PG(R);

- Free Space Path Loss : $$PL_{FS} = \frac{P_A}{P_R} = \left(\frac{4\pi R}{\lambda}\right)^2 \cdot \frac{1}{\rho_D \tau_P} \cdot \frac{1}{g_T g_R} = \frac{PL_{FS,ISO}}{g_T g_R}$$

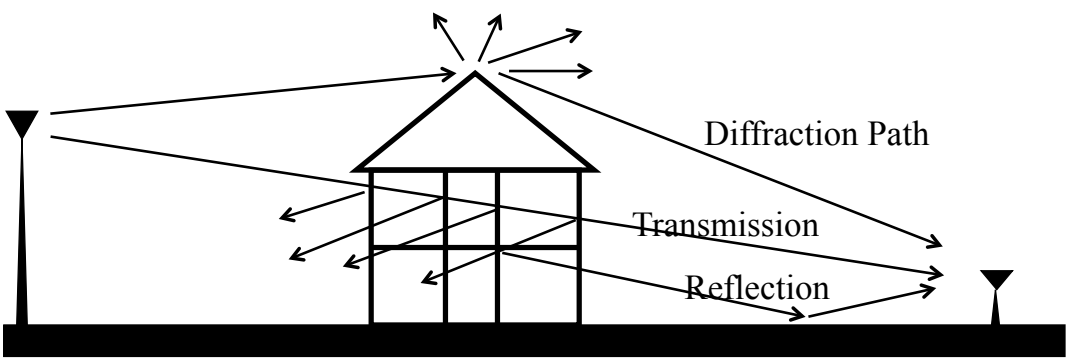
- Free Space Path Gain: $PG_{FS} = 1/PL_{FS}$ (decreases proportionally to $1/R^2$)

Propagation in presence of obstacles

The e.m. wave undergoes interactions with obstables before getting to the receiver. Interaction mechanisms can be classified into:

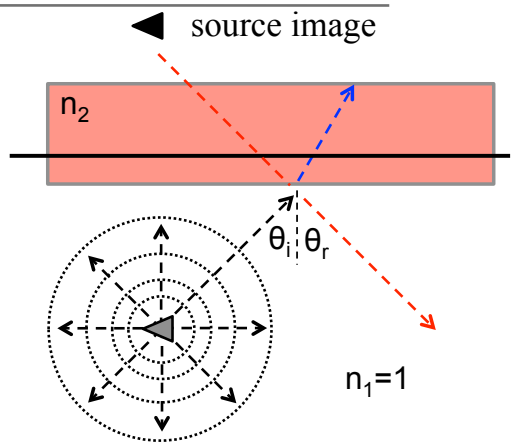
1) Reflection;
2) Transmission;
3) Diffraction;
4) Diffuse Scattering

Diffraction takes place due to the edge of a finite obstacle



Ray Reflection and Transmission

radial rays spring from the transmitting antenna

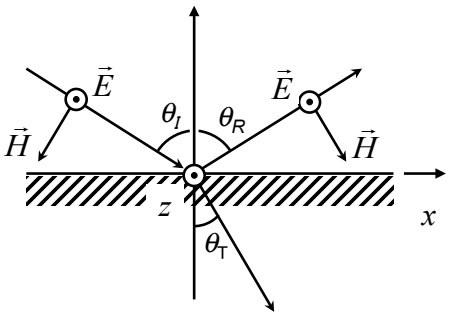


- when a ray impinges on the plane surface the corresponding wave is reflected and transmitted, thus generating **reflected** and **transmitted** rays

- The incident ray trajectory is modified according to the *Snell's laws of reflection (transmission)*. Reflected rays and wavefronts are *as if* the reflected wave were generated at the source image point…

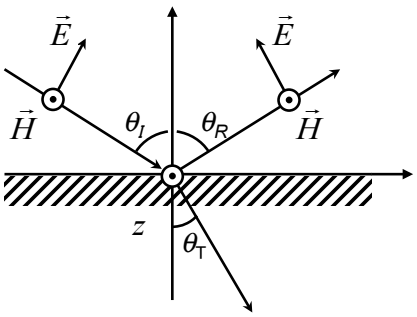
- The field amplitude / phase changes at the interaction point according to the Fresnel's *reflection (transmission) coefficients*

Reflection and Transmission Coefficients*



•TE polarization

$$\Gamma_{TE} = \frac{\cos\theta_i - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}} \; ; \; \tau_{TE} = 1 + \Gamma_{TE}$$



•TM polarization

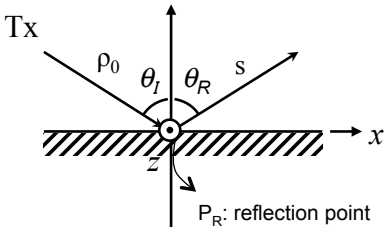
$$\Gamma_{TM} = \frac{\left(\frac{n_2}{n_1}\right)^2 \cos\theta_i - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}}{\left(\frac{n_2}{n_1}\right)^2 \cos\theta_i + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}} \; ; \; \tau_{TM} = 1 + \Gamma_{TM}$$

* Hp: plane waves, i.e. far from Tx

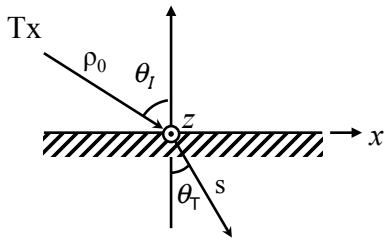
$n = \sqrt{\varepsilon_r}$ refractive index

Field formulation

Reflected ray



Transmitted ray



- trajectory: reflection law or Fermat's principle
- Field expression:

$$\begin{bmatrix} \vec{E}_r^{TE}(s) \\ \vec{E}_r^{TM}(s) \end{bmatrix} = \begin{bmatrix} \Gamma_{TE} & 0 \\ 0 & \Gamma_{TM} \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_i^{TE}(P_R) \\ \vec{E}_i^{TM}(P_R) \end{bmatrix} \cdot \frac{\rho_0}{\rho_0 + s} e^{-j\beta s}$$

Spreading factor

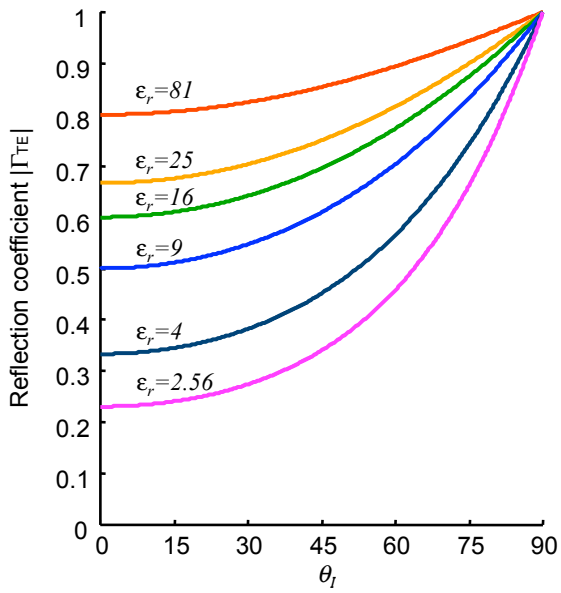
- direction: Snell's law or Fermat's principle
- Field expression:

$$\begin{bmatrix} \vec{E}_t^{TE}(s) \\ \vec{E}_t^{TM}(s) \end{bmatrix} = \begin{bmatrix} \tau_{TE} & 0 \\ 0 & \tau_{TM} \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_i^{TE}(P_R) \\ \vec{E}_i^{TM}(P_R) \end{bmatrix} \cdot \frac{\rho_0}{\rho_0 + s} e^{-j\beta' s}$$

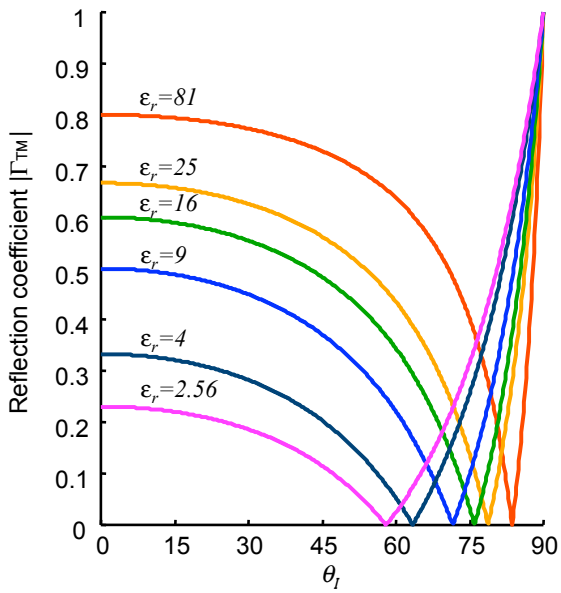
Fresnel's coefficients

Reflection does not change the spreading factor of the wave !!

Example: dielectric materials

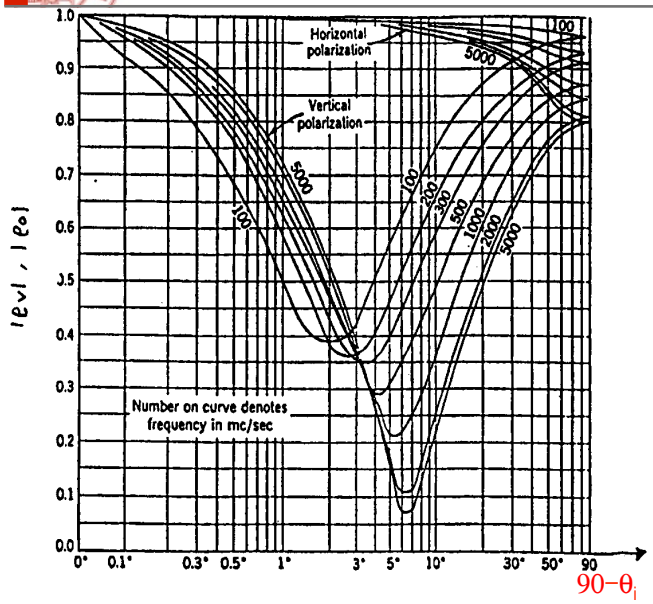


TE Polarization

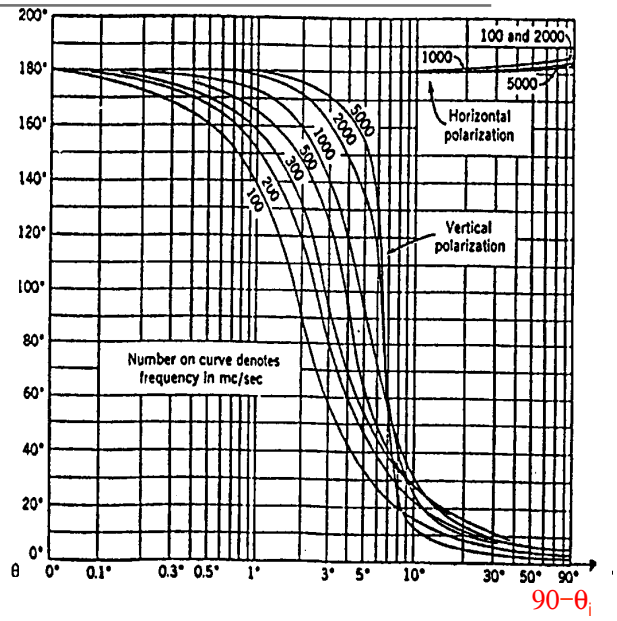


TM Polarization

Example: reflection on sea water



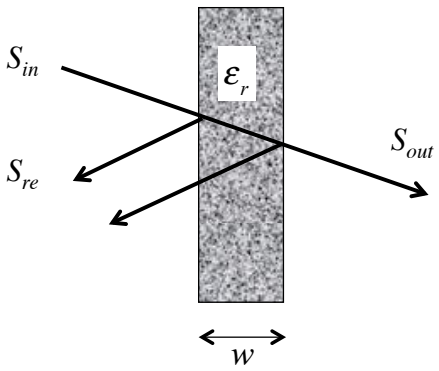
Module of G_{TE} and G_{TM}



Phase of G_{TE} and G_{TM}

Transmission through a wall (1/5)

* Hypotheses: - normal or quasi-normal incidence
- weakly lossy medium
- multiple reflections neglected



$$\Gamma \approx \Gamma_{TE} = \frac{\cos\theta_i - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}} \overset{\theta_i \simeq 0,\ n_1 = 1}{=} \frac{1-\sqrt{\varepsilon_r}}{1+\sqrt{\varepsilon_r}}$$

$$\frac{S_{re}}{S_{in}} = \frac{|E_{re}|^2 / 2\eta}{|E_{in}|^2 / 2\eta} = \frac{|E_{re}|^2}{|E_{in}|^2} \approx |\Gamma|^2$$

(Source: Prof. H.L. Bertoni)

Transmission through a wall (2/5)

In a lossy medium the wavenumber can be written as:

$$k = \omega\sqrt{\mu_0 \varepsilon_c} = \omega\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}$$

The complex relative dielectric constant can be written as:

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r'' = \frac{\varepsilon}{\varepsilon_0} - j\frac{\sigma}{\omega\varepsilon_0}$$

If the medium is weakly lossy ε" << ε' .
A plane wave propagating through the lossy medium has the expression:

$$\mathbf{E} = \mathbf{E_0} e^{-jkr} = \mathbf{E_0} e^{-(\alpha + j\beta)r}; \text{ with jk=}\alpha\text{+j}\beta$$

Thus:

$$k = \omega\sqrt{\mu_0 \varepsilon_0 \varepsilon_r} = \omega\sqrt{\mu_0 \varepsilon_0}\sqrt{\varepsilon_r' - j\varepsilon_r''} = \frac{\omega}{c}\sqrt{\varepsilon_r' - j\varepsilon_r''} =$$

$$= \frac{\omega}{c}\sqrt{\varepsilon_r'}\sqrt{1 - j\frac{\varepsilon_r''}{\varepsilon_r'}} \approx \frac{\omega}{c}\sqrt{\varepsilon_r'}\left(1 - j\frac{\varepsilon_r''}{2\varepsilon_r'}\right)$$

Where the series expansion has been truncated at first order

Transmission through a wall (3/5)

Therefore:

$$jk = \alpha + j\beta \approx \frac{\omega}{c}\sqrt{\varepsilon_r'}\left(\frac{\varepsilon_r''}{2\varepsilon_r'} + j\right) \Rightarrow$$

$$\begin{cases} \alpha \approx \dfrac{\omega}{c}\sqrt{\varepsilon_r'}\left(\dfrac{\varepsilon_r''}{2\varepsilon_r'}\right) = \dfrac{\omega}{c}\dfrac{\varepsilon_r''}{2\sqrt{\varepsilon_r'}} \\ \beta \approx \dfrac{\omega}{c}\sqrt{\varepsilon_r'} \end{cases}$$

$$\left|\bar{E}(r)\right| = \left|\bar{E}(0)\right| \cdot e^{-\alpha r}$$

$$S(r) = S(0) \cdot e^{-2\alpha r}$$

Transmission through a wall (4/5)

The reflection coefficient at normal incidence for the air-medium interface is

$$\Gamma_{0m} = \frac{1-\sqrt{\varepsilon_r}}{1+\sqrt{\varepsilon_r}}$$

The reflection coefficient for the second, medium-air interface is (see the expression of the reflection coefficients for normal incidence)

$$\Gamma_{m0} = \frac{\sqrt{\varepsilon_r}-1}{1+\sqrt{\varepsilon_r}} = -\Gamma_{0m}$$

Now if we consider the first interface we have

$$\frac{S_{refl1}}{S_{inc1}} = \frac{\left|\vec{E}_{refl1}\right|^2}{\left|\vec{E}_{in1}\right|^2} = \left|\Gamma_{0m}\right|^2$$

Transmission through a wall (5/5)

For power conservation we have:

$$S_{inc1} = S_{refl1} + S_{trasm1} \quad \Rightarrow \quad 1 = \frac{S_{refl1}}{S_{inc1}} + \frac{S_{trasm1}}{S_{inc1}} = \left|\Gamma_{0m}\right|^2 + \frac{S_{trasm1}}{S_{inc1}}$$

$$\frac{S_{trasm1}}{S_{inc1}} = 1 - \left|\Gamma_{0m}\right|^2$$

Now the transmitted power at the first interface, properly multiplied by the lossy-medium attenuation factor becomes the incident power S_{inc2} at the second interface, therefore we have

$$\frac{S_{refl2}}{S_{inc2}} = \left|\Gamma_{m0}\right|^2 = \left|\Gamma_{0m}\right|^2 = \left|\Gamma\right|^2 \quad ; \quad \frac{S_{transm2}}{S_{inc2}} = \frac{S_{transm2}}{S_{transm1}e^{-2\alpha w}} = \frac{S_{transm2}}{S_{inc1}\left(1-\left|\Gamma\right|^2\right)e^{-2\alpha w}} = 1-\left|\Gamma\right|^2$$

Thus:

$$\boxed{\frac{S_{transm2}}{S_{inc1}} = \frac{S_{out}}{S_{in}} = \left(1-\left|\Gamma\right|^2\right)^2 e^{-2\alpha w} \Rightarrow L_t = \frac{S_{in}}{S_{out}} = \frac{e^{2\alpha w}}{\left(1-\left|\Gamma\right|^2\right)^2}}$$

Example of Transmission Loss

Brick wall: $\varepsilon_r'=4$, $\varepsilon_r''=0.2$, $w=20$ cm

$$|\Gamma|^2 = \frac{S_{refl1}}{S_{inc1}} \approx \left|\frac{\sqrt{4}-1}{\sqrt{4}+1}\right|^2 = \frac{1}{9} = 0.11 \quad \text{or} \quad -9.6\text{dB}$$

at 1800 MHz (λ_o=1/6 m): $\alpha = \dfrac{0.2\pi}{(1/6)\sqrt{4}} = 1.88$

$$\mathrm{L}_t = \frac{S_{in}}{S_{out}} = (1-0.11)^{-2} e^{2(0.2)(1.88)} = 2.7 \quad \text{or} \quad 4.3\text{dB}$$

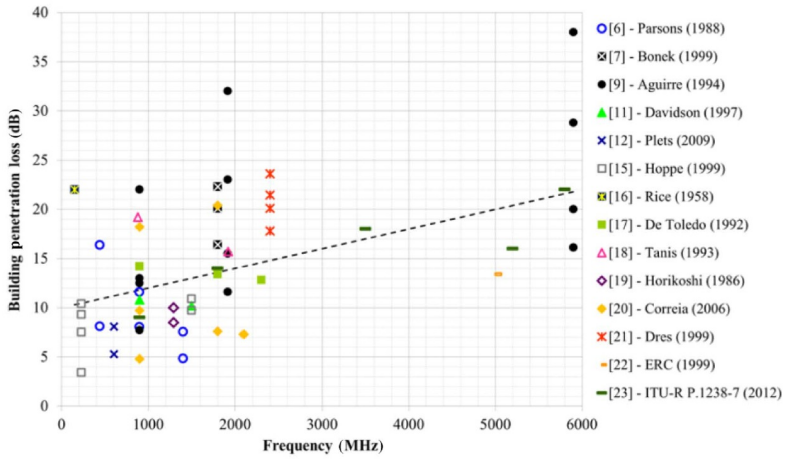
Building penetration loss (1/2)

- $\alpha = \dfrac{\omega}{c} \cdot \dfrac{\varepsilon_r''}{2\sqrt{\varepsilon_r'}} = \dfrac{\cancel{\omega}}{2c\sqrt{\varepsilon_r'}} \cdot \dfrac{\sigma}{\cancel{\omega}\varepsilon_0} = \dfrac{\sigma}{2\varepsilon_0 c\sqrt{\varepsilon_r'}}$

  Since ε_R is not very sensitive to frequency, whereas σ likely increases with frequency, larger transmission losses should be experienced at higher frequency;
- If outdoor-to-indoor propagation is now considered, building penetration losses (BPL) increasing with frequency should be expected. Actually, this is not clearly true, as shown by the BPL values shown in the following figure collected from the existing literature, where different and even somehow conflicting consideration can be found, e.g.:

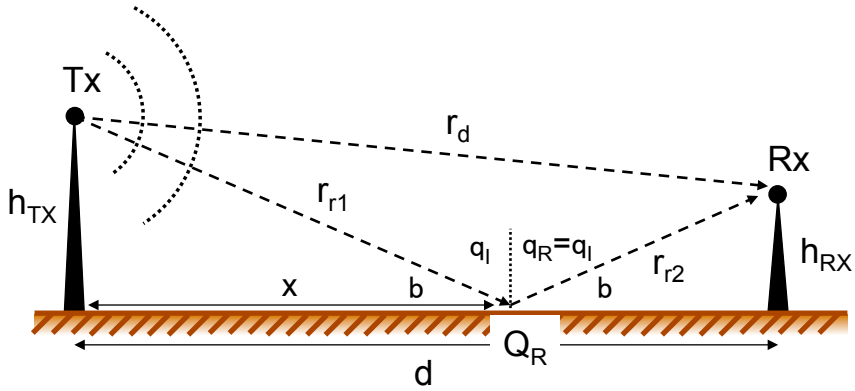
  - Prof. Bertoni:"over the frequency scale from under 100 MHz to 2 GHz, the penetration loss into office building is found to decrease with frequency";
  - Prof. Rappaport:"Several residential radio penetration studies show that penetration loss increases as the frequency increases";

Building penetration loss (2/2)



- The quite large variability highlighted in the figure is likely due to the fact that real building walls are never homogeneous, and they may have quite different inner structure and materials (brick walls are not exactly all the same!). The presence of windows, together with their number, size and position can also differently affect BPL values at different frequencies;

Ground effect – 2 ray model



$$r_d = \sqrt{d^2 + (h_{TX} - h_{RX})^2}$$

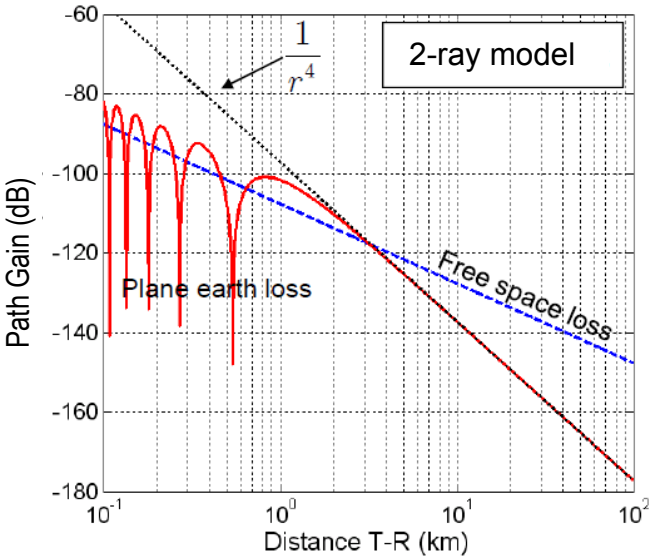
$$r_r = \sqrt{d^2 + (h_{TX} + h_{RX})^2}$$

- Total field at the Rx ($d >> h_{TX}, h_{RX}$) : $\vec{E}_{RX} \approx \frac{\vec{E}_0}{d} \cdot e^{-j\beta r_d} \cdot \left(1 + \Gamma \cdot e^{-j\beta \Delta r}\right)$; $\Delta r = r_r - r_d$

- Overall path gain

$$PG = \frac{P_R}{P_T} = (g_{Rv}\, g_{Tv}) \cdot \left(\frac{\lambda}{4\pi d}\right)^2 \cdot \overbrace{\left(1 + |\Gamma|^2 + 2 \cdot |\Gamma| \cdot \cos\left(\frac{4\pi}{\lambda}\frac{h_{TX} h_{RX}}{d} - \arg\Gamma\right)\right)}^{\text{correction factor w.r.t. free space}}$$

PG vs. distance



Quite different with respect to FS;

Physical reason: **interference** between the two received signal contributions;

After a breakpoint distance d_{BP}, interference is always destructive and the received power decays as $1/r^4$

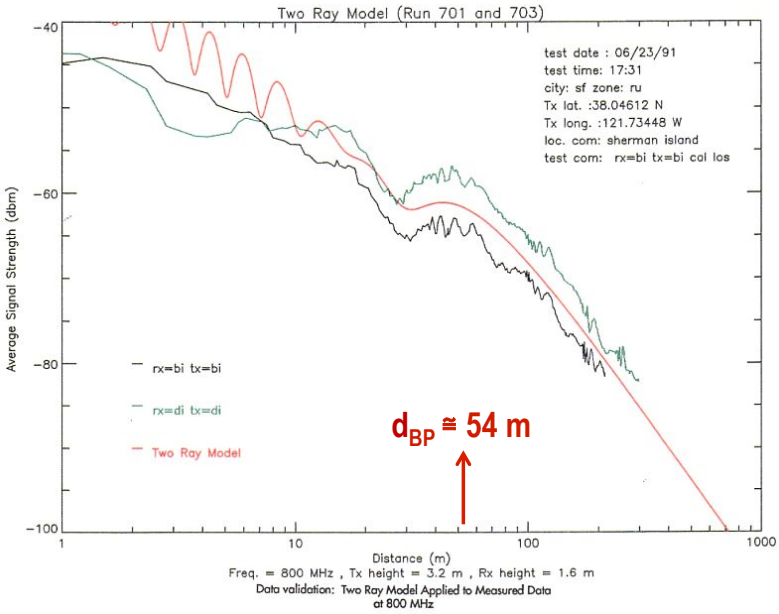
$$d_{BP} \approx \frac{4 \cdot h_{TX} \cdot h_{RX}}{\lambda}$$

Validation at Sherman Island

Sherman Island/Rural

August, 2011 © 2011 by H. L. Bertoni

(Source: Prof. H.L. Bertoni)

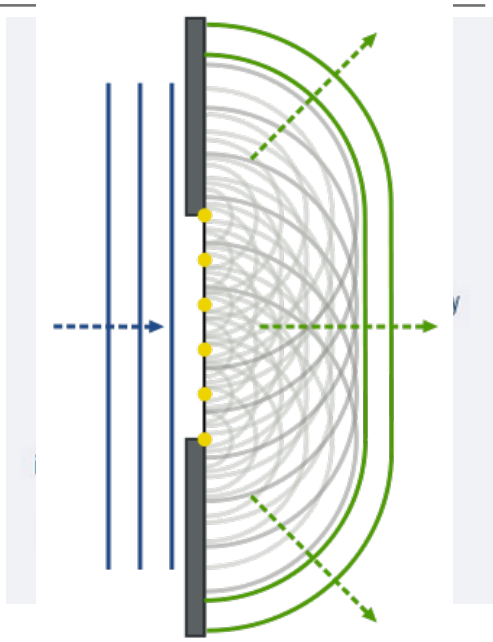


Introduction to diffraction phenomena

- First investigation on diffraction can be attributed to Fresnel, who at the beginning of 1800 exploited the 'secondary source principle' worked out by Huygens at the end of 1600 to account somehow for diffraction phenomena;
- In 1882 Kirchhoff 'put Fresnel's and Mawell's ideas together', i.e. he exploited Maxwell's equation to formalize and to corroborate the Fresnel's empirical theory;
- A geometrical theory of diffraction came quite later, due to the studies carried out mainly by Keller (1953) and Kouyoumjian and Pathak (1974);

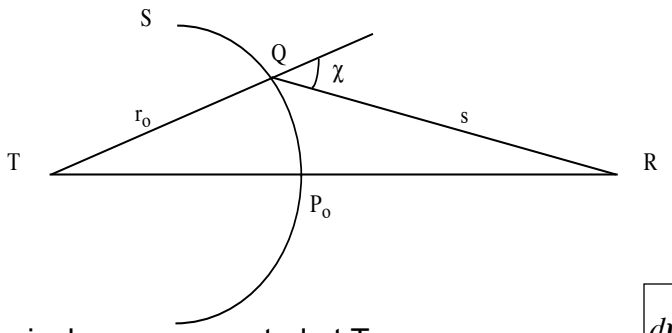
Diffraction - Huygens-Fresnel principle

- **Huygens principle**: given the wavefront S at t, it is possible to generate the wavefront S' at t+dt assuming that all surface elements dS of S become secondary point sources emitting spherical waves, whose **envelope** at t+dt constitutes S' at the same instant.



Huygens-Fresnel principle

➢ **Scalar theory:** Ψ generic field component



Spherical wave generated at T

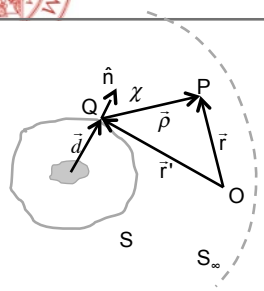
$$\psi_0(r_0) = A \cdot \frac{e^{-j\beta_0 r_0}}{r_0}$$

Field in R

$$d\psi(R) = K(\chi) \cdot A \cdot \frac{e^{-j\beta_0 r_0}}{r_0} \cdot \frac{e^{-j\beta_0 s}}{s} \cdot dS$$

$$\psi(R) = \int_{wavefront\ S} K(\chi) \cdot A \cdot \frac{e^{-j\beta_0 r_0}}{r_0} \cdot \frac{e^{-j\beta_0 s}}{s}\, \mathrm{d}S$$

Kirchhoff's theorem (1/2)



➢ In a homogeneous, source-free region we have:

$$\nabla^2\Psi - \sigma^2\Psi = 0 \xrightarrow{\text{Green's function method}} \Psi(\vec{r}) = \int_S \left(\Psi \cdot \frac{\partial G}{\partial n} - G \cdot \frac{\partial \Psi}{\partial n} \right) dS$$

in this case

$$\Psi(\vec{r}) = -\int_{S \cup S_\infty} \left(\Psi \cdot \frac{\partial G}{\partial n} - G \cdot \frac{\partial \Psi}{\partial n} \right) \mathrm{d}S$$

➢Hypotheses:

- d , ρ >> λ
- loss-less medium: σ=jβ
- $\lim_{r\to\infty} r \cdot \left| \frac{\partial \Psi}{\partial n} \right| = \lim_{r\to\infty} r \cdot |\Psi| = 0$
- $G(\rho) = -\frac{1}{4\pi} \frac{e^{-j\beta\rho}}{\rho}$ Green's function
- S = wavefront

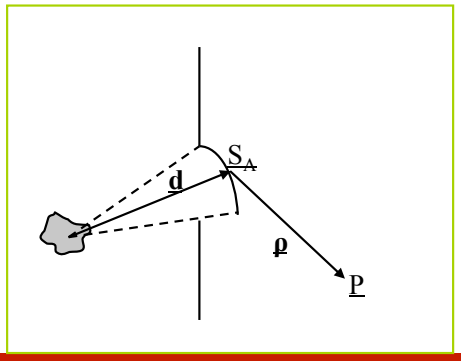
$$\Psi(\vec{r}) = \frac{j\beta}{4\pi} \int_S \overbrace{F(\vartheta,\phi) \cdot \frac{e^{-j\beta d}}{d}}^{field\ \Psi\ in\ Q} \cdot \frac{e^{-j\beta\rho}}{\rho} \cdot (1 + \cos\chi)\, \mathrm{d}S$$

$$K(\chi) = \frac{j\beta}{4\pi} \cdot (1 + \cos\chi)$$

Kirchhoff's theorem (2/2)

- In free-space Kirchhoff theorem is valid but irrelevant because the free space formula can be used

- Kirchhoff theorem becomes useful to determine the field in presence of an obstacle. The integral must be limited to the free portion of the wavefront:

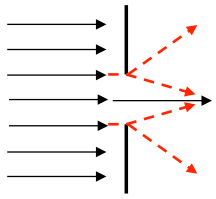


$$\Psi(\vec{r}) = \frac{j\beta}{4\pi} \int_{S_A} F(\vartheta,\phi) \cdot \frac{e^{-j\beta(d+\rho)}}{d \cdot \rho} \cdot (1+\cos\chi)\, \mathrm{d}S$$

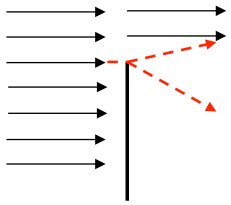
Ψ on S_A can be approximated with its value in absence of the obstacle (Kirchhoff's approximation);

Diffraction fundamental examples

Diffraction examples:



Aperture diffraction



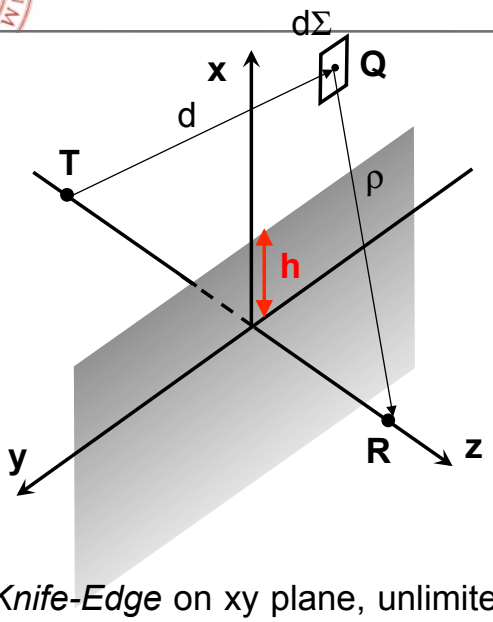
"Knife Edge" diffraction

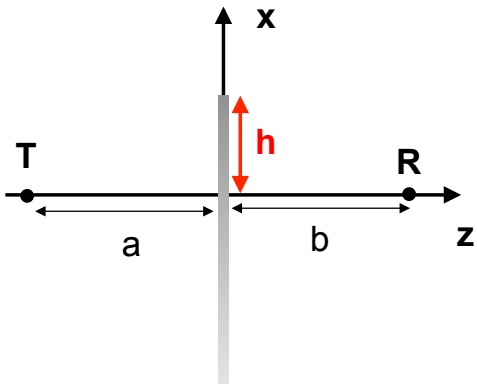
- In particular:
  - The field is non zero even in the shadow region;
  - The field is perturbated elsewhere

- Diffraction is stronger when lambda is large w.r.t. the obstacle

- Kirchoff theorem allows scalar diffraction field computation in virtually every problem, by solving an integral

Knife-edge diffraction (1/4)





- *Knife-Edge* on xy plane, unlimited in y and limited between h e -∞ in the x direction;
- T and R on the z axis on opposite sides w.r.t. the obstacle

Knife-edge diffraction (2/4)

- From dΣ element of S_A

$$dE(R) = \frac{j\beta}{4\pi} \cdot F(\vartheta,\phi) \cdot \frac{e^{-j\beta d}}{d} \cdot \frac{e^{-j\beta\rho}}{\rho} \cdot (1+\cos\chi)\, d\Sigma$$

- Hp 1: far source so that the wavefrom is plane and coincides with plane xy

$$E(R) = \frac{j\beta}{4\pi} \cdot \int_{-\infty}^{+\infty}\int_{h}^{+\infty} F(\vartheta,\phi) \cdot \frac{e^{-j\beta d}}{d} \cdot \frac{e^{-j\beta\rho}}{\rho} \cdot (1+\cos\chi)\, dx\, dy$$

$$E(R) = \frac{j\beta}{4\pi} \cdot \frac{e^{-j\beta(a+b)}}{ab} \int_{-\infty}^{+\infty}\int_{h}^{+\infty} F(\vartheta,\phi) \cdot \frac{e^{-j\beta(d-a)}}{d} a \cdot \frac{e^{-j\beta(\rho-b)}}{\rho} b \cdot (1+\cos\chi)\, dx\, dy$$

- Hp 2: h << a,b
- Hp 3: the only relevant secondary sources dΣ have x << a,b ; y << a,b

Knife-edge diffraction (3/4)

Hp2 + Hp3

$F(\theta,\phi) = A$ constant; $\chi \cong 0$

$d \approx a$, $\rho \approx b$

We assume $d = a$, $\rho = b$

For the exponents we have

$$d - a = \sqrt{(x^2 + y^2) + a^2} - a = a\sqrt{1 + \frac{(x^2 + y^2)}{a^2}} - a$$

$$\sqrt{1+z} \underset{z \to 0}{\approx} 1 + \frac{z}{2}$$

$$d - a \approx a\left[1 + \frac{(x^2 + y^2)}{2a^2}\right] - a = \frac{(x^2 + y^2)}{2a}$$

$$\rho - b \approx \frac{(x^2 + y^2)}{2b} \qquad \text{(similarly)}$$

Knife-edge diffraction (4/4)

$$E(R) = \frac{j\beta}{2\pi} \cdot A \cdot \frac{e^{-j\beta(a+b)}}{ab} \int_{-\infty}^{+\infty} \int_{h}^{+\infty} e^{-j\beta \frac{x^2+y^2}{2a}} \cdot e^{-j\beta \frac{x^2+y^2}{2b}} \, dx\, dy$$

$$= \frac{j\beta}{2\pi} \cdot A \cdot \frac{e^{-j\beta(a+b)}}{ab} \int_{-\infty}^{+\infty} \int_{h}^{+\infty} e^{-j\beta \frac{a+b}{2ab}(x^2+y^2)} \, dx\, dy$$

Knife-edge attenuation (1/3)

- $E_0(R)$ = field in absence of the obstacle

$$E_0(R) = \frac{j\beta}{2\pi} \cdot A \cdot \frac{e^{-j\beta(a+b)}}{ab} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-j\beta\frac{a+b}{2ab}\left(x^2+y^2\right)}\, dx\, dy$$

- Diffraction attenuation $\mathbf{L_S}$

$$\sqrt{L_S} = \left|\frac{E_0}{E}\right| = \left|\frac{\frac{j\beta}{2\pi} \cdot A \cdot \frac{e^{-j\beta(a+b)}}{ab} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-j\beta\frac{a+b}{2ab}\left(x^2+y^2\right)}\, dx\, dy}{\frac{j\beta}{2\pi} \cdot A \cdot \frac{e^{-j\beta(a+b)}}{ab} \int_{-\infty}^{+\infty}\int_{h}^{+\infty} e^{-j\beta\frac{a+b}{2ab}\left(x^2+y^2\right)}\, dx\, dy}\right| = \left|\frac{\int_{-\infty}^{+\infty} e^{-j\beta\frac{a+b}{2ab}x^2}\, dx}{\int_{h}^{+\infty} e^{-j\beta\frac{a+b}{2ab}x^2}\, dx}\right|$$

$$\sqrt{L_S} = 2 \cdot \left|\frac{\int_{0}^{+\infty} e^{-j\beta\frac{a+b}{2ab}x^2}\, dx}{\int_{h}^{+\infty} e^{-j\beta\frac{a+b}{2ab}x^2}\, dx}\right|$$

Knife-edge attenuation (2/3)

- If: $\nu = x \cdot \sqrt{\frac{2}{\lambda}\frac{a+b}{ab}} \quad \Rightarrow dx = \frac{d\nu}{\sqrt{\frac{2}{\lambda}\frac{a+b}{ab}}}$

$$\sqrt{L_S} = \left|\frac{E_0}{E}\right| = \left|\frac{\frac{2}{\sqrt{\frac{2}{\lambda}\frac{a+b}{ab}}} \int_{0}^{+\infty} e^{-j\frac{\pi}{2}\nu^2}\, d\nu}{\frac{1}{\sqrt{\frac{2}{\lambda}\frac{a+b}{ab}}} \int_{h\cdot\sqrt{\frac{2}{\lambda}\frac{a+b}{ab}}}^{+\infty} e^{-j\frac{\pi}{2}\nu^2}\, d\nu}\right| = \left|\frac{2\int_{0}^{+\infty} e^{-j\frac{\pi}{2}\nu^2}\, d\nu}{\int_{h\cdot\sqrt{\frac{2}{\lambda}\frac{a+b}{ab}}}^{+\infty} e^{-j\frac{\pi}{2}\nu^2}\, d\nu}\right|$$

$$\frac{1}{\sqrt{2}} \cdot e^{-j\frac{\pi}{4}} = \frac{1}{2}(1-j)$$

Fresnel's integral

ν_0: Fresnel's parameter

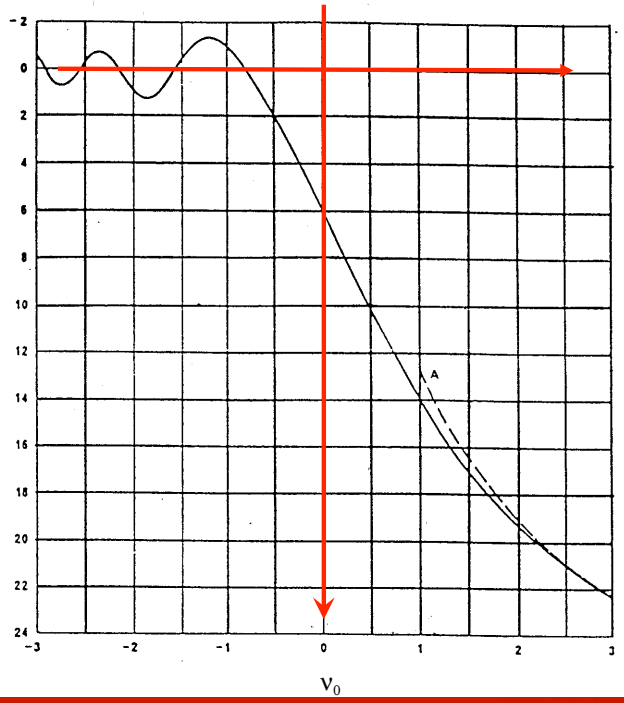
$$\boxed{\sqrt{L_S} = \left|\frac{E_0}{E}\right| = \frac{1}{\left|\frac{1+j}{2}\int_{\nu_0}^{+\infty} e^{-j\frac{\pi}{2}\nu^2}\, d\nu\right|} = \frac{1}{\frac{1}{\sqrt{2}}\left|\int_{\nu_0}^{+\infty} e^{-j\frac{\pi}{2}\nu^2}\, d\nu\right|}}$$

with:

$$\nu_o = h \cdot \sqrt{\frac{2}{\lambda}\frac{a+b}{ab}}$$

Knife-edge attenuation (3/3)

L_S (dB) = 20 log (E_0/ E)



- For $v_0 < \approx -\sqrt{2}$ $|L_S(dB)|$ is lower than 1 dB, therefore diffraction attenuation is negligible .

Approximation for $v_0 > -0.78$ [*]

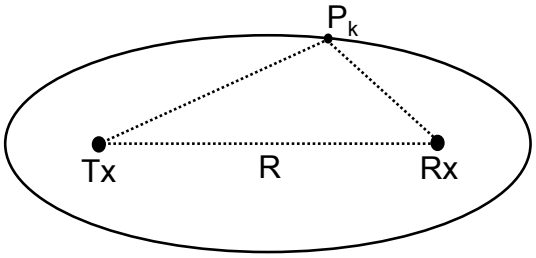
$$L_S(dB) = 6.9 + 20 \cdot \log_{10}\left(\sqrt{(v_0 - 0.1)^2 + 1} + v_0 - 0.1\right)$$

$$L(v_0) = \begin{cases} -20\log\left(0.5 - 0.62 v_0\right) & -0.8 < v_0 < 0 \\ -20\log\left[0.5\exp\left(-0.95 v_0\right)\right] & 0 < v_0 < 1 \\ -20\log\left[0.4 - \left\{0.1184 - \left(0.38 - 0.1 v_0\right)^2\right\}^{1/2}\right] & 1 < v_0 < 2.4 \\ -20\log\left[\dfrac{0.225}{v_0}\right] & v_0 > 2.4 \end{cases}$$

[*] ITU-R recommendation P.526-13 “Propagation by diffraction”
[**]W. C. Y. Lee, Mobile Communications Engineering, Mc Graw Hill, New York 1982

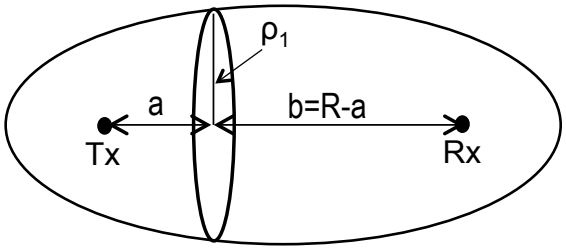
The (First) Fresnel ellipsoid (1/2)

- Definition: $|TxP_k| + |RxP_k| = R + \frac{\lambda}{2}$



- Fresnel radius: $\rho_1 = \sqrt{\lambda \cdot \frac{a \cdot b}{a+b}}$

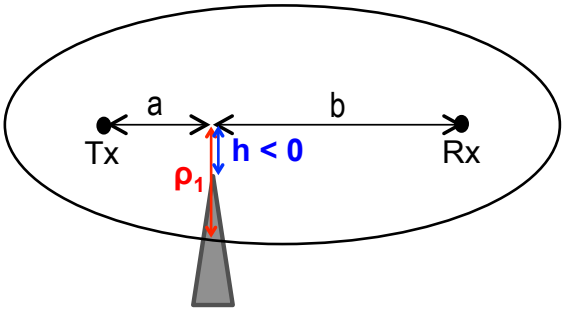
$$\left(\rho_{1,max} \underset{a=b=R/2}{=} \frac{\sqrt{\lambda \cdot R}}{2} \right)$$

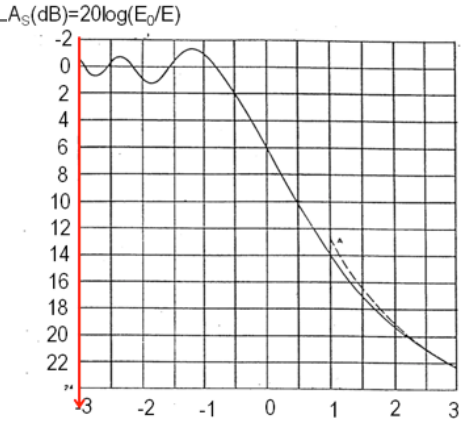


- In LoS condition, objects placed outside the Fresnel ellipsoid (also known as Fresnel zone) may generate new rays, but they have only small effect on the direct ray field ➔ objects outside the Fresnel ellipsoid are often neglected, since they generate negligible effects

The (First) Fresnel ellipsoid (2/2)

- Example: knife-edge inside the Fresnel ellipsoid





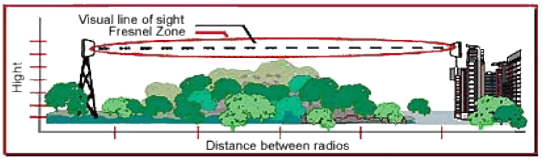
In order that the presence of the KE produces negligible effect, $\nu < -\sqrt{2}$ must be required (additional loss lower than 1 dB)

$$\nu < -\sqrt{2} \Rightarrow h < -\sqrt{\lambda \frac{a \cdot b}{a+b}} = -\rho_1 \Rightarrow |h| > \rho_1$$

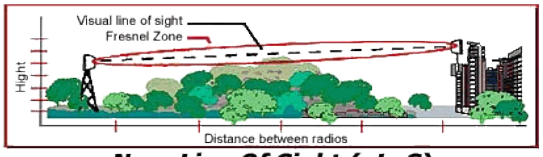
The KE must be outside the Fresnel ellipsoid !!

Fresnel zone clearance

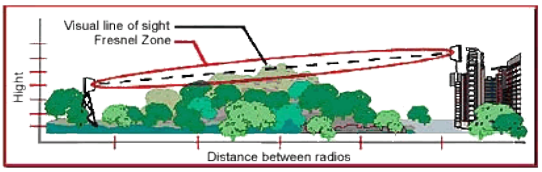
- In the design of point-to-point wireless link, the clearance of the first Fresnel zone is pursued through a proper choice of the communication frequency and of the positioning of both the transmitter and the receiver;
- In case no objects are present inside the Fresnel zone, propagation approximately occurs as in free space;



Line Of Sight (LoS)



Near Line Of Sight (nLoS)



Non Line Of Sight (NLoS)

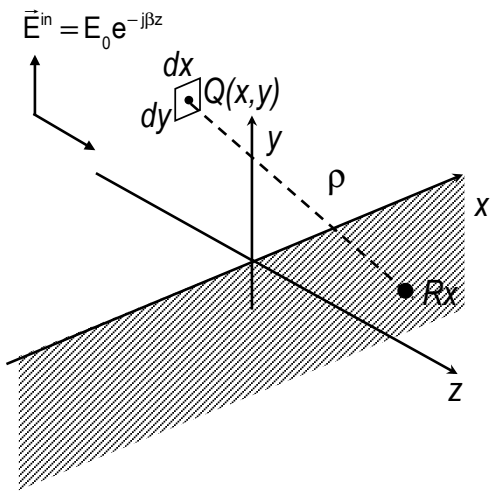
KE and Diffracted Rays (1/3)

- Assuming a plane wave illumination in the KE problem, the FK integral can be solved and the field beyond the KE can be computed in closed-form[*]

$$E_{Rx} = A\frac{j\beta}{4\pi}\int_0^{\infty}\int_{-\infty}^{\infty}(1+\cos\chi)\cdot\frac{e^{-j\beta\rho}}{\rho}dydx$$

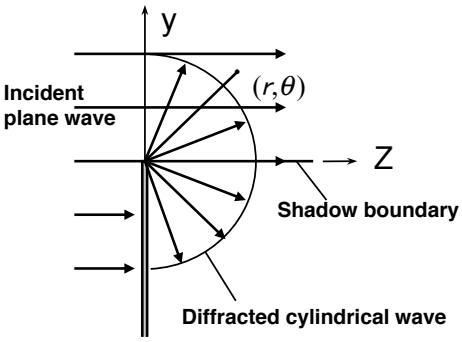
Stationary phase method

$$E_{Rx} = A_0\cdot e^{-j\beta x}\cdot U(\theta) + A_0\cdot e^{-j\pi/4}\cdot\frac{e^{-j\beta\rho}}{\sqrt{\rho}}\cdot D(\theta)$$



[*] H. L. Bertoni, *Radio Propagation for Modern Wireless Systems*, Prentice Hall 2000

KE and Diffracted Rays (2/3)



$$E_{Rx} = \underbrace{A_0 \cdot e^{-j\beta z} \cdot U(\theta)}_{\substack{\text{incident plane wave} \\ (\text{for } \theta>0)}} + \underbrace{A_0 \cdot e^{-j\pi/4} \cdot \frac{e^{-j\beta\rho}}{\sqrt{\rho}} \cdot D(\theta)}_{\text{cilindrical diffracted wave}}$$

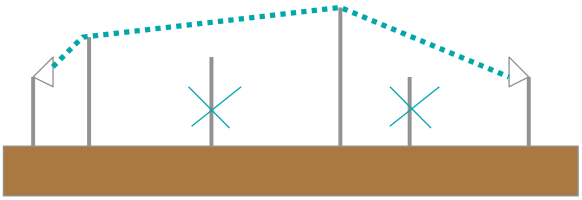
- A diffracted wave is generated by the presence of the knife-edge and under the hypothesis made it is a cylindrical wave .
- The wave surfaces are therefore cylinders having the axis coincident with the edge of the knife-edge ➔ thus we can define the *Diffracted Rays* propagating from the obstacle edge in the radial direction.
- $D(\theta) = \frac{-1}{\sqrt{2\pi\beta}} \cdot \frac{1+\cos\theta}{2\sin\theta}$: Diffraction coefficient

Multiple "knife-edge" diffraction

- While a closed-form expression for single knife edge diffraction is available, it is not so for multiple knife edge diffraction. There are solutions for two knife edges (Millington) but only iterative solutions are available for a higher number of k.e.'s.
- Therefore **heuristic** methods have been developed which consist of arbitrary geometric constructions conceived so as to resort to multiple computations of single-knife-edge diffraction losses

The rubber-band/Epstein-Peterson method

- The *rubber-band method* is a profile-simplification method:
  - ***An ideal elastic band is stretched over the link profile. Only those knife-edges that are touched by the band are selected, the others are dropped***

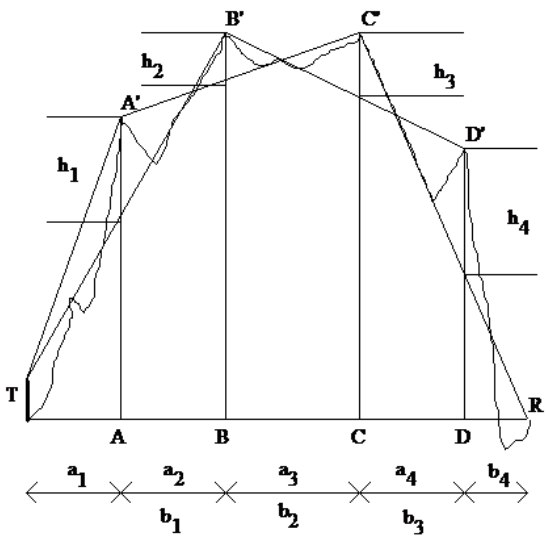


- The E.P method is based on a decomposition of the path in sub-paths each one experiencing only one knife-edge diffraction

- The excess loss is computed as a product of each single sub-path loss. The E.P method is not very accurate for a number of obstacles > 4-5

The Epstein-Peterson method (II)

A partial path is associated to each obstacle which spans from the preceding to the following obstacles (virtual Tx and Rx, respectively)



$$\sqrt{L_{S\_TOT}} = \prod_{i=1}^{n_o} \left| \frac{1}{\frac{1+j}{2}\int_{v_{0i}}^{+\infty} e^{-j\frac{\pi}{2}v^2}\, dv} \right|$$

v_{0i} is the Fresnel's parameter for the i-th obstacle

$$\mathbf{v_{0i} = h_i \sqrt{\frac{2}{\lambda}\frac{a_i + b_i}{a_i b_i}} \quad i = 1,...N}$$

h_i a_i b_i are shown in the figure. Notice that the h_i values are measured from the Tx-Rx line of sight

The Saunders and Bonar method (outline)

It's the combination of two different methods
Flat edge method: computes the loss due to a uniform series of knife-edges **Voegler's method[*]** : allows the computation of the loss due to an arbitrary series of knife-edges (limited number of k.e.'s for CPU time reasons) using a recursive algorithm

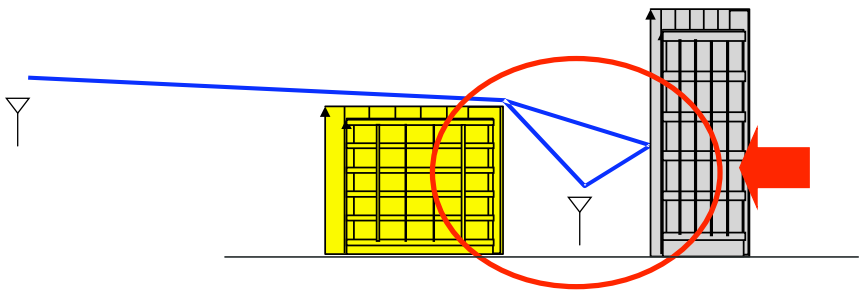
At first the loss L_1 due to a mean, uniformized profile is computed with the Flat Edge method. Then the original profile is simplified (e.g. with the tight rope method) reducing it to a low number of knife edges. Thus the loss L_2 is computed and the final excess loss is obtained as a combination of L_1 and L_2

The Saunders and Bonar method, although quite complex is the most accurate heuristic multi knife-edge method.

[*] L. E. Voegler, "An attenuation function for multiple knife-edges diffraction," Radio Sci., Vol. 17, No. 6, pp. 1541-1546, 1982

Additional notes on ORT methods

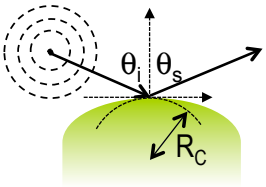
- Multi knife-edge methods are often used for Over-Roof-Top (ORT) propagation prediction
- **ORT** models only predict multiple diffraction loss along the radial
- **Roof-to-street** propagation, including reflections etc. is not included
- Specific ray models for roof-to-street propagation have been developed

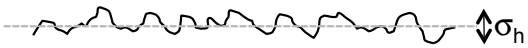


Interaction with real objects

- Reflection/Refraction/Diffraction modeling traditionally refers to *flat*, *smooth*, *homogeneous* half-space boundary or layer with *infinite* extension. These assumptions are often unsuited to represent real obstacles

- Nevertheless, the results achieved for the ideal representation still hold for real objects provided that:
  - curvature radius (R_C) much larger than λ;
  - 'roughness degree' much smaller than λ; Rayleigh criterion: $\sigma_h < \lambda/(8\cos\theta_i)$
  - linear extension much larger than λ;



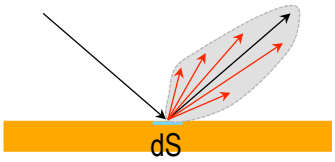


Otherwise, the object behaves as **diffusive scatterer**, i.e. it spreads the impinging energy in all directions.

Scatterer / Reflector

- Real objects are quite often neither ideal reflector nor ideal scatterer: the impinging power on a surface element dS is partly reflected in the specular direction, partly scattered in different directions.

- A scattering lobe steered in the direction of specular reflection can be often hypothesized;



Radar Cross Section

- The overall scattering properties (including reflection and diffraction) of an object can be accounted by its *Radar Cross Section* (RCS):

$$\sigma(\theta,\phi) = 4\pi r^2 \frac{S_{scat}(r,\theta,\phi)}{S_{inc}} = 4\pi r^2 \frac{\left|\vec{E}_{scat}(r,\theta,\phi)\right|^2}{\left|\vec{E}_{inc}\right|^2} \quad [m^2]$$

$$\sigma_T = \frac{1}{4\pi} \cdot \int_{4\pi} \sigma(\theta_s,\phi_s)\, d\Omega = \frac{P_{scat,tot}}{S_{inc}}$$

- Example: in the framework of military application, stealth technology aims at designing aircrafts with an RCS as low as possible, in order to make them undetectable by radar (Sukhoi T-50 seems to have the same RCS of a tennis ball)
- According to its definition, RCS is often computed through the direct measurement of the scattered field. Only in few, ideal cases, expressions of the RCS as a function of the object geometrical and electromagnetic properties are available.

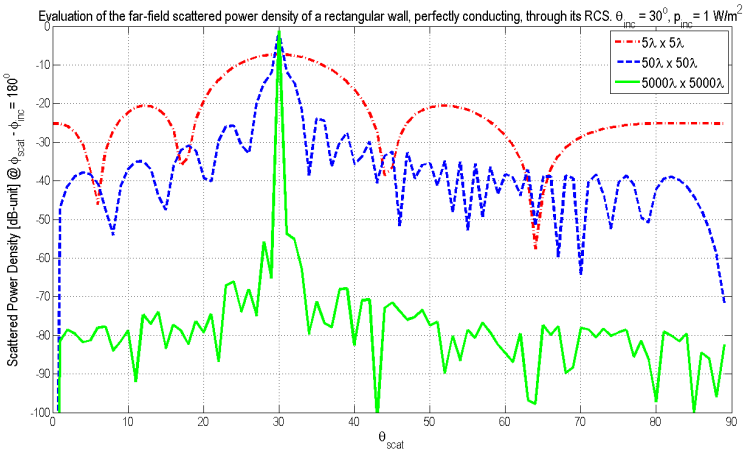
RCS of a metallic plate

- Example: rectangular metallic flat, smooth plate of size a, b:

$$\sigma(\theta,\phi) = 4\pi \cdot \left(\frac{ab}{\lambda}\right)^2 \cdot \left(\cos^2\theta_i \cdot \cos^2\theta \cdot \cos^2\phi + \sin^2\phi\right) \cdot \mathrm{sinc}^2\left(\frac{X}{\pi}\right) \cdot \mathrm{sinc}^2\left(\frac{Y}{\pi}\right)$$

$$X = \beta \cdot \frac{a}{2} \cdot \sin\theta \cdot \cos\phi$$

$$Y = \beta \cdot \frac{b}{2} \cdot \sin\theta \cdot \sin\phi - \mathrm{sir}$$

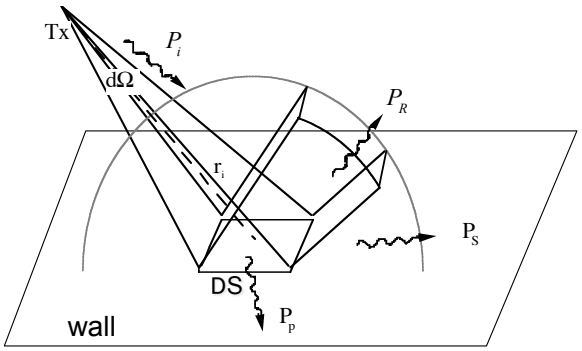


The 'Effective Roughness' model

- It's a scalar, incoherent model aiming at simply but effectively describe diffuse scattering (DS) from a surface element dS;

- the power density p of the scattered field is proportional to a *scattering coefficient* S, related to the overall amount of diffuse power, and is modeled according to a proper *scattering pattern*, describing its spatial distribution;

- DS occurs at the expenses of specular reflection and transmission, and therefore a proper reduction factor (0<R<1) must be applied to properly reduce reflection/ transmission coefficients, in order to satisfy the overall power balance



The 'Effective Roughness' model

- Several studies have proved that a single-lobe pattern centered on the direction of the specular reflection/transmission is often the best representation;

$$\begin{cases} p(\psi_R) \propto S_R^2 \cdot \left(\dfrac{1+\cos\psi_R}{2}\right)^{\alpha_R} \\ p(\psi_T) \propto S_T^2 \cdot \left(\dfrac{1+\cos\psi_T}{2}\right)^{\alpha_T} \end{cases}$$

- S_R/S_T: scattering coefficient for the backward/ forward half space;
- ψ_R/ψ_T: angle between the reflection/transmission direction and the scattering direction
- ψ_R/ψ_T sets the width of the scattering lobe the backward/forward half space;

- At UHF frequencies, S ranges from 0.2 to 0.4 in rural environment, while values up to 0.6, have been estimated in more complex scenarios; Typical values of α range between 2 and 4

Further reading:
V. Degli-Esposti, F. Fuschini, E. Vitucci, G. Falciasecca, "Measurement and modeling of scattering from buildings", IEEE Transactions on Antennas and Propagation, Vol. 55 No 1, pp. 143-153, January 2007