ESoA course on
Large Scale Radio Propagation

General Theory of Propagation

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Radiation in free space

Far field: \( r \gg \lambda \quad r >> D, \text{antenna size} \)

\[ E(r,\theta,\varphi) \approx E_o(\theta,\varphi) \frac{e^{-j\beta r}}{r} \]

vectorial amplitude \( \overrightarrow{E_o} \perp \overrightarrow{r} \)
propagation factor \( \beta = \frac{2\pi}{\lambda} \)

\( \vec{E} \quad \text{Complex vector or phasor, so that} \)
\[ \vec{e}(r,\theta,\varphi,t) = \Re \{ \vec{E}(r,\theta,\varphi)e^{j\omega t} \} \]
Detailed far field expressions

\[
\vec{E}(r, \theta, \varphi) \approx -j \frac{\eta}{2\lambda} \left[ \hat{i}_r \times \vec{M}(\theta, \varphi) \times \hat{i}_r \right] \frac{e^{-j\beta r}}{r}
\]

\[
\vec{H}(r, \theta, \varphi) \approx \frac{1}{\eta} \hat{i}_r \times \vec{E}
\]

\[
\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^*
\]

Pointing vector

Antenna’s radiation vector

\[
\vec{M}(\theta, \varphi) \approx \int_{V'} \vec{j}(P') \exp[j\beta (\vec{r}', \hat{i}_r)] dV'
\]

Polarization vector

\[
\hat{p} \equiv \frac{\vec{E}}{|\vec{E}|} \cdot e^{j\chi}
\]

The polarization vector defines the polarization of the field (and also of the antenna)

It is strictly related to the radiation vector \( \vec{M} \)

\[
\hat{p} = \frac{\vec{E}}{|\vec{E}|} \cdot e^{j\chi} = \frac{-j \eta}{2\lambda r} \frac{\vec{M} \ e^{-j\beta r}}{r} \cdot e^{j\chi} = \frac{\vec{M} \ e^{-j(\beta r + \pi/2)}}{|\vec{M}|} \cdot e^{j\chi} \Rightarrow \hat{p} = \frac{\vec{M} \ e^{-j(\beta r + \pi/2)}}{|\vec{M}|} = \hat{M} \]
**Emitted field**

The field emitted by an antenna can be derived from the transmitted power $P_T$ and the polarization vector using the following formula

$$
\vec{E} = \sqrt{\frac{\eta P_T g_T}{2\pi}} \frac{e^{-j\beta r}}{r} \hat{p} = \sqrt{60P_T g_T} \frac{e^{-j\beta r}}{r} \hat{p}
$$

**Wavefront**

A *wavefront* is a locus where the field has constant phase.

$$
E_x(r, \theta, \phi) = E_{0x}(\theta, \phi) \frac{e^{-j\beta r}}{r} = \frac{|E_{0x}|}{r} e^{j(\text{arg}(E_{0x}) - \beta r)}
$$

Therefore the wavefront is defined by:

$$
\text{arg}(E_{0x}) - \beta r = -\bar{\phi}
$$

If $r$ is large and arg$(E_{0x})$ is small we have

$$
\text{arg}(E_{0x}) - \beta r \approx -\bar{\phi} \Rightarrow r = \frac{\bar{\phi}}{\beta}
$$

Therefore the wavefront is a spherical surface. That’s why it’s called “spherical wave”.
Spherical and plane waves

Spherical wave

\[ \vec{E}(r, \theta, \varphi) \approx \vec{E}_o(\theta, \varphi) \frac{e^{-j\beta r}}{r} \]

In far field a spherical wave can be locally approximated with a Plane wave

\[ \vec{E}(r, \theta, \varphi) \approx \vec{E}_o' e^{-j \vec{k} \cdot \vec{r}} \]

with

\[ \vec{E}_o' = \frac{\vec{E}_o(\theta, \varphi)}{r} \]

\[ \vec{k} = \beta \hat{r} \]

Definition of Ray (1/2)

- Given a propagating wave, every curve that is everywhere perpendicular to the wavefront is called electromagnetic ray. A ray is the path of the wavefront. There is a mutual identification btw wave and ray.

- Therefore we assume that the ray also “has” a field or “carries” a field, the field of the corresponding wave at every point.

E.g: spherical wave and rays generated by a Tx antenna.
Definition of Ray (2/2)

- In free space, rays are rectilinear.
- In presence of concentrated obstacles rays are piecewise-rectilinear and wavefronts can be of various kinds (see further on).
- In non-homogeneous media rays can be curved (not treated here).

Ex.1 Spherical wave and rectilinear rays
Ex.2 reflected spherical wave and piece-wise rectilinear rays

Poynting vector and power-density

The Poynting vector always has the same direction as the ray. For a spherical wave it is easy to show:

\[
\mathbf{S} = \frac{1}{2} \mathbf{E} \times \mathbf{H}^* = \frac{1}{2} \mathbf{E} \times \left( \frac{1}{\eta} \mathbf{i}_r \times \mathbf{E}^* \right) = \frac{1}{2\eta} \mathbf{i}_r \mathbf{E}^2,
\]

where we used the identity:

\[
\mathbf{A} \times \mathbf{B} \times \mathbf{C} = (\mathbf{A} \cdot \mathbf{C}) \mathbf{B} - (\mathbf{A} \cdot \mathbf{B}) \mathbf{C}
\]

Therefore:
e.m. power propagates along rays!

radiation Power-Density or Intensity is defined as:

\[
|\mathbf{S}| = S = \frac{\mathbf{E}^2}{2\eta}
\]
Ray tube and power-density law

Def: a Ray tube (tube of flux) is a closed tube whose lateral surface is formed by a set of rays and the bases by two wavefront sections.

By applying the Poynting’s theorem (power balance) to a ray tube having bases $d\Sigma_1$, $d\Sigma_2$ small enough to assume the field constant on them, and considering a lossless medium we have:

$$\oint_{\Sigma} \bar{S} \cdot \hat{n} \; d\Sigma = \int_{d\Sigma_1} \bar{S} \cdot \hat{n} \; d\Sigma + \int_{d\Sigma_2} \bar{S} \cdot \hat{n} \; d\Sigma = 0$$

$$= -|\bar{S}_1| \cdot d\Sigma_1 + |\bar{S}_2| \cdot d\Sigma_2 = 0 \quad \Rightarrow \quad S_1 \cdot d\Sigma_1 = S_2 \cdot d\Sigma_2 \quad \Rightarrow \quad \frac{S_2}{S_1} = \frac{d\Sigma_1}{d\Sigma_2}$$

Power-density (or Intensity) law of Geometrical Optics:
“Power-density is inversely proportional to the cross-section of the ray tube”

Spreading Factor

We can therefore define the spreading or divergence factor $A$:

$$A = \sqrt{\frac{S_2}{S_1}} = \frac{|E_2|}{|E_1|} = \sqrt{\frac{d\Sigma_1}{d\Sigma_2}}$$

The spreading factor accounts for the field attenuation due to the enlarging of the ray- tube cross-section.
The field carried by a ray attenuates even in lossless media because the power spreads over an enlarging wavefront surface as the wave propagates.
Re-writing the spherical wave

Using a reference distance $\rho_0$, we have

\[
E(R) = E_0 e^{-j\beta R} = E_0 \frac{\rho_0}{R} e^{-j\beta (R-\rho_0)} = E(\rho_0) \frac{\rho_0}{R} e^{-j\beta (R-\rho_0)} = \frac{E_0}{R} e^{-j\beta R}
\]

Field:

\[
E(\rho_0) \frac{\rho_0}{R} e^{-j\beta (R-\rho_0)} = E(\rho_0) \cdot \frac{\rho_0}{\rho_0 + s} e^{-j\beta s}
\]

"Divergence factor"

\[
\text{Power density: } S(R) = S(\rho_0) \left(\frac{\rho_0}{\rho_0 + s}\right)^2
\]

Spreading factor

Received power $P_r$ is proportional to power density, that’s why $P_r$ attenuates with the square of distance!

\[
P_r = S \cdot A_{\text{eff}} ; \quad S \triangleq \frac{|E|^2}{2\eta}
\]

Free space radio link

- The simplest example of wireless channel is represented by the “Free Space” (FS) case; the Tx and the Rx are completely surrounded by an homogeneous, isotropic, linear and loss-less medium. Antennas are in Line of Sight (LoS) and propagation can be limited to the only direct ray;

- The radio link can be fully described by the Friis (or Free Space) formula:

\[
P_R = P_A g_T(\theta_T, \phi_T) g_R(\theta_R, \phi_R) \left(\frac{\lambda}{4\pi R}\right)^2 \rho_D \tau_p
\]

"inverse power law"

- $R$: link distance
- $P_A$: power absorbed (radiated) by the transmitting antenna;
- $g_T, g_R$: antenna gains;
- $\lambda$: wavelength
- $\rho_D$: power matching factor;
- $\tau_p$: polarization matching factor
Path Loss and Path Gain

- **Path Gain (PG)**: describes how the received power (field strength) decreases with link distance \( \Rightarrow PG = PG(R) \);
- **Path Loss (PL)**: is the reciprocal of PG, i.e. it describes how the attenuation of the propagating signal increases with link distance \( \Rightarrow PL = PL(R) = 1/PG(R) \);

Free Space Path Loss:

\[
PL_{FS} = \frac{P_A}{P_R} = \left( \frac{4\pi R}{\lambda} \right)^2 \cdot \frac{1}{\rho_D \tau_p} \cdot \frac{1}{gTgR} = \frac{PL_{FS,ISO}}{gTgR}
\]

Free Space Path Gain: \( PG_{FS} = 1/PL_{FS} \) (decreases proportionally to \( 1/R^2 \))

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Propagation in presence of obstacles

The e.m. wave undergoes interactions with obstacles before getting to the receiver. Interaction mechanisms can be classified into:

1) Reflection;
2) Transmission;
3) Diffraction;
4) Diffuse Scattering

Diffraction takes place due to the edge of a finite obstacle
Ray Reflection and Transmission

- when a ray impinges on the plane surface the corresponding wave is reflected and transmitted, thus generating reflected and transmitted rays

- The incident ray trajectory is modified according to the Snell’s laws of reflection (transmission). Reflected rays and wavefronts are as if the reflected wave were generated at the source image point...

- The field amplitude / phase changes at the interaction point according to the Fresnel’s reflection (transmission) coefficients

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Reflection and Transmission Coefficients*

*Hp: plane waves, i.e. far from Tx

\[ n = \sqrt{\varepsilon_r} \] refractive index

• **TE polarization**

\[
\Gamma_{TE} = \frac{\cos \theta_i - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2 \theta_i}}; \quad \tau_{TE} = 1 + \Gamma_{TE}
\]

• **TM polarization**

\[
\Gamma_{TM} = \frac{\left(\frac{n_2}{n_1}\right)^2 \cos \theta_i - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2 \theta_i}}{\left(\frac{n_2}{n_1}\right)^2 \cos \theta_i + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2 \theta_i}}; \quad \tau_{TM} = 1 + \Gamma_{TM}
\]
Field formulation

- trajectory: reflection law or Fermat’s principle
- Field expression:

\[
\begin{bmatrix}
\tilde{E}_t^{\text{TE}}(s) \\
\tilde{E}_t^{\text{TM}}(s)
\end{bmatrix} =
\begin{bmatrix}
\Gamma_{\text{TE}} & 0 \\
0 & \Gamma_{\text{TM}}
\end{bmatrix}
\begin{bmatrix}
\tilde{E}_r^{\text{TE}}(p_R) \\
\tilde{E}_r^{\text{TM}}(p_R)
\end{bmatrix}
\cdot 
\frac{\rho_0}{\rho_0 + s}
\cdot e^{-j\beta s}
\]

- direction: Snell’s law or Fermat’s principle
- Field expression:

\[
\begin{bmatrix}
\tilde{E}_t^{\text{TE}}(s) \\
\tilde{E}_t^{\text{TM}}(s)
\end{bmatrix} =
\begin{bmatrix}
\tau_{\text{TE}} & 0 \\
0 & \tau_{\text{TM}}
\end{bmatrix}
\begin{bmatrix}
\tilde{E}_i^{\text{TE}}(p_R) \\
\tilde{E}_i^{\text{TM}}(p_R)
\end{bmatrix}
\cdot 
\frac{\rho_0}{\rho_0 + s}
\cdot e^{-j\beta s}
\]

Reflection does not change the spreading factor of the wave !!

Example: dielectric materials

| $\varepsilon_r$ | Reflection coefficient $|\Gamma_{\text{TE}}|$ | Reflection coefficient $|\Gamma_{\text{TM}}|$ |
|---|---|---|
| 81 | 0.3 | 0.1 |
| 25 | 0.5 | 0.3 |
| 16 | 0.6 | 0.4 |
| 9 | 0.7 | 0.5 |
| 4 | 0.8 | 0.6 |
| 2.56 | 0.9 | 0.7 |

TE Polarization | TM Polarization
Example: reflection on sea water

Module of $G_{TE}$ and $G_{TM}$

Phase of $G_{TE}$ and $G_{TM}$

Transmission through a wall (1/5)

* Hypotheses: - normal or quasi-normal incidence
  - weakly lossy medium
  - multiple reflections neglected

$$\Gamma \approx \Gamma_{TE} = \frac{\cos \theta_i - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2 \theta_i}} = \frac{1 - \sqrt{\varepsilon_r}}{1 + \sqrt{\varepsilon_r}}$$

$$\frac{S_{re}}{S_{in}} = \left|\frac{E_{re}}{E_{in}}\right|^2 = \left|\frac{E_{re}}{E_{in}}\right|^2$$

(Source: Prof. H.L. Bertoni)
Transmission through a wall (2/5)

In a lossy medium the wavenumber can be written as:

\[ k = \omega \sqrt{\mu_0 \varepsilon_r} = \omega \sqrt{\mu_0 \varepsilon_0 \varepsilon_r} \]

The complex relative dielectric constant can be written as:

\[ \varepsilon_r = \varepsilon_r' - j \varepsilon_r'' = \frac{\varepsilon}{\varepsilon_0} - j \frac{\sigma}{\omega \varepsilon_0} \]

If the medium is weakly lossy, \( \varepsilon'' \ll \varepsilon' \).

A plane wave propagating through the lossy medium has the expression:

\[ E = E_0 e^{-jkr} = E_0 e^{-(\alpha + j\beta)r} \text{; with } jk = \alpha + j\beta \]

Thus:

\[ k = \omega \sqrt{\mu_0 \varepsilon_0 \varepsilon_r} = \omega \sqrt{\mu_0 \varepsilon_0} \sqrt{\varepsilon_r' - j \varepsilon_r''} = \frac{\omega}{c} \sqrt{\varepsilon_r' - j \varepsilon_r''} = \frac{\omega}{c} \sqrt{\varepsilon_r' \left(1 - \frac{j \varepsilon_r''}{2 \varepsilon_r'}\right)} \]

Where the series expansion has been truncated at first order

Transmission through a wall (3/5)

Therefore:

\[ jk = \alpha + j\beta = \frac{\omega}{c} \sqrt{\varepsilon_r'} \left(\frac{\varepsilon_r''}{2 \varepsilon_r'} + j\right) \Rightarrow \]

\[ \begin{aligned} 
\alpha &\approx \frac{\omega}{c} \sqrt{\varepsilon_r'} \left(\frac{\varepsilon_r''}{2 \varepsilon_r'}\right) = \frac{\omega}{c} \frac{\varepsilon_r''}{2 \varepsilon_r'} \\
\beta &\approx \frac{\omega}{c} \sqrt{\varepsilon_r'} 
\end{aligned} \]

\[ |E(r)| = |E(0)| e^{-\alpha r} \]

\[ S(r) = S(0) \cdot e^{-2\alpha r} \]
The reflection coefficient at normal incidence for the air-medium interface is
\[ \Gamma_{0m} = \frac{1 - \sqrt{\varepsilon_r}}{1 + \sqrt{\varepsilon_r}} \]

The reflection coefficient for the second, medium-air interface is (see the expression of the reflection coefficients for normal incidence)
\[ \Gamma_{m0} = \frac{\sqrt{\varepsilon_r} - 1}{1 + \sqrt{\varepsilon_r}} = -\Gamma_{0m} \]

Now if we consider the first interface we have
\[ \frac{S_{\text{refl}1}}{S_{\text{inc}1}} = \left| \frac{\tilde{E}_{\text{refl}1}}{\tilde{E}_{\text{in}1}} \right|^2 = |\Gamma_{0m}|^2 \]

For power conservation we have:
\[ S_{\text{inc}1} = S_{\text{refl}1} + S_{\text{transm}1} \Rightarrow 1 = \frac{S_{\text{refl}1}}{S_{\text{inc}1}} + \frac{S_{\text{transm}1}}{S_{\text{inc}1}} = |\Gamma_{0m}|^2 + \frac{S_{\text{transm}1}}{S_{\text{inc}1}} \]
\[ \frac{S_{\text{transm}1}}{S_{\text{inc}1}} = 1 - |\Gamma_{0m}|^2 \]

Now the transmitted power at the first interface, properly multiplied by the lossy-medium attenuation factor becomes the incident power \( S_{\text{inc}2} \) at the second interface, therefore we have
\[ \frac{S_{\text{refl}2}}{S_{\text{inc}2}} = |\Gamma_{m0}|^2 = |\Gamma_{0m}|^2 = |\Gamma|^2 ; \quad \frac{S_{\text{transm}2}}{S_{\text{inc}2}} = \frac{S_{\text{transm}2}}{S_{\text{transm}1}} e^{-2\alpha w} = \frac{S_{\text{transm}2}}{S_{\text{inc}1} (1 - |\Gamma|^2)} e^{-2\alpha w} = 1 - |\Gamma|^2 \]

Thus:
\[ \frac{S_{\text{transm}2}}{S_{\text{inc}1}} = \frac{S_{\text{out}}}{S_{\text{in}}} = \left( 1 - |\Gamma|^2 \right)^2 e^{-2\alpha w} \Rightarrow L_t = \frac{S_{\text{in}}}{S_{\text{out}}} = \frac{e^{2\alpha w}}{\left( 1 - |\Gamma|^2 \right)^2} \]

For power conservation we have:
\[ S_{\text{inc}2} = S_{\text{refl}2} + S_{\text{transm}2} \Rightarrow 1 = \frac{S_{\text{refl}2}}{S_{\text{inc}2}} + \frac{S_{\text{transm}2}}{S_{\text{inc}2}} = |\Gamma|^2 + \frac{S_{\text{transm}2}}{S_{\text{inc}2}} \]
\[ \frac{S_{\text{transm}2}}{S_{\text{inc}2}} = 1 - |\Gamma|^2 \]

Now the transmitted power at the first interface, properly multiplied by the lossy-medium attenuation factor becomes the incident power \( S_{\text{inc}2} \) at the second interface, therefore we have
\[ \frac{S_{\text{refl}2}}{S_{\text{inc}2}} = |\Gamma_{m0}|^2 = |\Gamma_{0m}|^2 = |\Gamma|^2 ; \quad \frac{S_{\text{transm}2}}{S_{\text{inc}2}} = \frac{S_{\text{transm}2}}{S_{\text{transm}1}} e^{-2\alpha w} = \frac{S_{\text{transm}2}}{S_{\text{inc}1} (1 - |\Gamma|^2)} e^{-2\alpha w} = 1 - |\Gamma|^2 \]

Thus:
\[ \frac{S_{\text{transm}2}}{S_{\text{inc}1}} = \frac{S_{\text{out}}}{S_{\text{in}}} = \left( 1 - |\Gamma|^2 \right)^2 e^{-2\alpha w} \Rightarrow L_t = \frac{S_{\text{in}}}{S_{\text{out}}} = \frac{e^{2\alpha w}}{\left( 1 - |\Gamma|^2 \right)^2} \]
Example of Transmission Loss

Brick wall: $\varepsilon_r'=4, \varepsilon_r''=0.2, w=20$ cm

$$|\Gamma|^2 = \frac{S_{\text{refl}}}{S_{\text{inc}}} \approx \left| \frac{\sqrt{4}-1}{\sqrt{4}+1} \right|^2 = \frac{1}{9} = 0.11 \text{ or } -9.6\text{dB}$$

at 1800 MHz ($\lambda_o=1/6$ m): $\alpha = \frac{0.2\pi}{(1/6)\sqrt{4}} = 1.88$

$$L_i = \frac{S_{\text{in}}}{S_{\text{out}}} = (1-0.11)^2 e^{2(0.2)(1.88)} = 2.7 \text{ or } 4.3\text{dB}$$

Building penetration loss (1/2)

- $\alpha = \frac{\omega}{c} \cdot \frac{\varepsilon_r''}{2\sqrt{\varepsilon_r'}} = \frac{\sigma}{2c\sqrt{\varepsilon_r'}}$, $\frac{\sigma}{\sigma_0} = \frac{\sigma}{2\varepsilon_0 c\sqrt{\varepsilon_r'}}$

Since $\varepsilon_R$ is not very sensitive to frequency, whereas $\sigma$ likely increases with frequency, larger transmission losses should be experienced at higher frequency;

- If outdoor-to-indoor propagation is now considered, building penetration losses (BPL) increasing with frequency should be expected. Actually, this is not clearly true, as shown by the BPL values shown in the following figure collected from the existing literature, where different and even somehow conflicting consideration can be found, e.g.:
  - Prof. Bertoni: "over the frequency scale from under 100 MHz to 2 GHz, the penetration loss into office building is found to decrease with frequency";
  - Prof. Rappaport: "Several residential radio penetration studies show that penetration loss increases as the frequency increases";
Building penetration loss (2/2)

• The quite large variability highlighted in the figure is likely due to the fact that real building walls are never homogeneous, and they may have quite different inner structure and materials (brick walls are not exactly all the same!). The presence of windows, together with their number, size and position can also differently affect BPL values at different frequencies;

Ground effect – 2 ray model

• Total field at the Rx ($d >> h_{TX}, h_{RX}$):

\[ \vec{E}_{RX} \approx \frac{\vec{E}_0}{d} \cdot e^{-j\beta d} \left( 1 + \Gamma \cdot e^{-j\Delta r(\Gamma)} \right) ; \quad \Delta r = r_r - r_d \]

• Overall path gain

\[ PG = \frac{P_R}{P_T} = \left( g_R \cdot g_{TV} \right) \cdot \left( \frac{\lambda}{4\pi d} \right)^2 \cdot \left[ 1 + |\Gamma|^2 + 2 \cdot |\Gamma| \cdot \cos \left( \frac{4\pi h_{TX} h_{RX}}{\lambda d} - \arg \Gamma \right) \right] \]
PG vs. distance

Quite different with respect to FS;
Physical reason: interference between the two received signal contributions;
After a breakpoint distance $d_{BP}$, interference is always destructive and the received power decays as $1/r^4$

$$d_{BP} \approx \frac{4 \cdot h_{TX} \cdot h_{RX}}{\lambda}$$

Validation at Sherman Island

(Source: Prof. H.L. Bertoni)
Introduction to diffraction phenomena

• First investigation on diffraction can be attributed to Fresnel, who at the beginning of 1800 exploited the ‘secondary source principle’ worked out by Huygens at the end of 1600 to account somehow for diffraction phenomena;
• In 1882 Kirchhoff ‘put Fresnel’s and Mawell’s ideas together’, i.e. he exploited Maxwell’s equation to formalize and to corroborate the Fresnel's empirical theory;
• A geometrical theory of diffraction came quite later, due to the studies carried out mainly by Keller (1953) and Kouyoumjian and Pathak (1974);

Diffraction - Huygens-Fresnel principle

- **Huygens principle**: given the wavefront $S$ at $t$, it is possible to generate the wavefront $S’$ at $t+dt$ assuming that all surface elements $dS$ of $S$ become secondary point sources emitting spherical waves, whose envelope at $t+dt$ constitutes $S’$ at the same instant.
Huygens-Fresnel principle

**Scalar theory:** \[ \Psi \] generic field component

\[
\psi_0(r_0) = A \cdot e^{-j\beta r_0}
\]

Spherical wave generated at T

Field in R

\[
d\psi(R) = K(\chi) \cdot A \cdot \frac{e^{-j\beta r_0}}{r_0} \cdot \frac{e^{-j\beta s}}{s} \cdot dS
\]

\[
\psi(R) = \int_{\text{wavefront } S} K(\chi) \cdot A \cdot \frac{e^{-j\beta r_0}}{r_0} \cdot \frac{e^{-j\beta s}}{s} \cdot dS
\]

Kirchhoff's theorem (1/2)

- In a homogeneous, source-free region we have:

\[ \nabla^2 \Psi - \sigma^2 \Psi = 0 \]

\[ \Psi(\bar{r}) = \int_S \left( \Psi \cdot \frac{\partial G}{\partial n} - G \cdot \frac{\partial \Psi}{\partial n} \right) dS \]

in this case

\[ \Psi(\bar{r}) = - \int_{S_{\text{field}}} \left( \Psi \cdot \frac{\partial G}{\partial n} - G \cdot \frac{\partial \Psi}{\partial n} \right) dS \]

**Hypotheses:**
- \( d, \rho >> \lambda \)
- Loss-less medium: \( \sigma = j\beta \)
- \( \lim_{r \to \infty} \frac{\partial \Psi}{\partial n} = \lim_{r \to \infty} r \cdot |\Psi| = 0 \)
- \( G(\rho) = -\frac{1}{4\pi} \frac{e^{-j\beta \rho}}{\rho} \) Green's function
- \( S = \text{wavefront} \)

\[ \Psi(\bar{r}) = \frac{jB}{4\pi} \int_S F(\theta,\phi) \cdot \frac{e^{-j\beta d}}{d} \cdot \frac{e^{-j\beta \rho}}{\rho} \cdot (1 + \cos \chi) \cdot dS \]

\[ K(\chi) = \frac{jB}{4\pi} \cdot (1 + \cos \chi) \]
Kirchhoff’s theorem (2/2)

- In free-space Kirchhoff theorem is valid but irrelevant because the free space formula can be used.

- Kirchhoff theorem becomes useful to determine the field in presence of an obstacle. The integral must be limited to the free portion of the wavefront:

\[
\Psi(\vec{r}) = \frac{j\beta}{4\pi} \int_{S_A} F(\vartheta, \phi) \cdot e^{-j\beta(d+\rho)} \cdot \frac{1}{d \cdot \rho} \cdot (1 + \cos \chi) \, dS
\]

\(\Psi\) on \(S_A\) can be approximated with its value in absence of the obstacle (Kirchhoff’s approximation);

Diffraction fundamental examples

Diffraction examples:

- Aperture diffraction
- "Knife Edge" diffraction

- In particular:
  - The field is non zero even in the shadow region;
  - The field is perturbated elsewhere

- Diffraction is stronger when lambda is large w.r.t. the obstacle

- Kirchoff theorem allows scalar diffraction field computation in virtually every problem, by solving an integral
Knife-edge diffraction (1/4)

- **Knife-Edge** on xy plane, unlimited in y and limited between h e -∞ in the x direction;
- T and R on the z axis on opposite sides w.r.t. the obstacle

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Knife-edge diffraction (2/4)

- From dΣ element of Sₐ
  \[
  dE(R) = \frac{j\beta}{4\pi} \cdot F(\vartheta, \phi) \cdot e^{-j\beta d \over d} \cdot e^{-j\beta \rho \over \rho} \cdot (1 + \cos \chi) d\Sigma
  \]

- Hp 1: far source so that the wavefrom is plane and coincides with plane xy
  \[
  E(R) = \frac{j\beta}{4\pi} \int_{-\infty}^{+\infty} \int_{-h}^{+h} F(\vartheta, \phi) \cdot e^{-j\beta d \over d} \cdot e^{-j\beta \rho \over \rho} \cdot (1 + \cos \chi) dx dy
  \]

- Hp 4: h << a,b
- Hp 3: the only relevant secondary sources dΣ have
  \[
  x << a,b \ ; \ y << a,b
  \]
Knife-edge diffraction (3/4)

\[ F(\theta, \phi) = A \text{ constant; } \chi \equiv 0 \]
\[ d \approx a, \rho \approx b \]

We assume \( d = a, \rho = b \)

For the exponents we have

\[ d - a = \sqrt{\left( k^2 + y^2 \right) + a^2} - a = a \sqrt{1 + \frac{k^2 + y^2}{a^2}} - a \]

\[ \sqrt{1 + z} \approx 1 + \frac{z}{2} \quad z \to 0 \]

\[ d - a \approx a \left[ 1 + \frac{k^2 + y^2}{2a^2} \right] - a = \frac{k^2 + y^2}{2a} \]

\[ \rho - b \approx \frac{k^2 + y^2}{2b} \quad (\text{similarly}) \]

---

Knife-edge diffraction (4/4)

\[ E(R) = \frac{jB}{2\pi} \cdot A \cdot \frac{e^{-jB(a+b)}}{ab} \int_{-\infty}^{\infty} dx \int_{-h}^{\infty} e^{-jBx^2 + y^2} \cdot e^{-jBx^2 + y^2} dy \]

\[ = \frac{jB}{2\pi} \cdot A \cdot \frac{e^{-jB(a+b)}}{ab} \int_{-\infty}^{\infty} dx \int_{-h}^{\infty} e^{-jBx^2 + y^2} e^{-jBx^2 + y^2} dx dy \]
Knife-edge attenuation (1/3)

- $E_0(R) = \text{field in absence of the obstacle}$
  \[ E_0(R) = \frac{j\beta}{2\pi} \cdot A \cdot e^{-j\beta(a+b)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-j\frac{\alpha+\beta}{2ab}(x^2+y^2)} \, dx \, dy \]

- Diffraction attenuation $L_S$
  \[ \sqrt{L_S} = \frac{|E_0|}{E} = \frac{j\beta}{2\pi} \cdot A \cdot e^{-j\beta(a+b)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-j\frac{\alpha+\beta}{2ab}(x^2+y^2)} \, dx \, dy \]
  \[ \Rightarrow \sqrt{L_S} = 2 \cdot \frac{e^{\frac{-\beta}{2\lambda^2}}}{\lambda^2} \]

Knife-edge attenuation (2/3)

- If: $v = x \cdot \sqrt{\frac{a+b}{\lambda \cdot ab}} \Rightarrow dx = \frac{dv}{\sqrt{\frac{2 a + b}{\lambda \cdot ab}}}$
  \[ \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{4}} = \frac{1}{2} (1-i) \]

- $L_S = 2 \cdot \frac{e^{\frac{-\beta}{2\lambda^2}}}{\lambda^2} \]

- Fresnel’s integral

- $v_0$: Fresnel’s parameter

- With:
  \[ v_0 = h \cdot \sqrt{\frac{2 a + b}{\lambda \cdot ab}} \]
Knife-edge attenuation (3/3)

For $\nu_0 \leq -\sqrt{2}$, $|L_S(dB)|$ is lower than 1 dB, therefore diffraction attenuation is negligible.

Approximation for $\nu_0 > -0.78$ [*]

$L_S(dB) = 20 \log (E_0/E)$

$\nu_0$ = 

-0.8< $\nu_0 < 0$

$-20 \log \left( 0.5 - 0.62 \nu_0 \right)$

$0 < \nu_0 < 1$

$-20 \log \left[ 0.5 \exp \left( -0.95 \nu_0 \right) \right]$  

$1 < \nu_0 < 2.4$

$-20 \log \left[ \frac{0.225}{\nu_0} \right]$  

$\nu_0 > 2.4$

$L(v_0) = \begin{cases} 
-20 \log \left( 0.5 - 0.62 v_0 \right), & -0.8 < v_0 < 0 \\
-20 \log \left[ 0.5 \exp \left( -0.95 v_0 \right) \right], & 0 < v_0 < 1 \\
-20 \log \left[ 0.4 - \left( 0.1184 - \left( 0.38 - 0.1 v_0 \right)^2 \right)^{1/2} \right], & 1 < v_0 < 2.4 \\
-20 \log \left[ \frac{0.225}{v_0} \right], & v_0 > 2.4 
\end{cases}$

[*] ITU-R recommendation P.526-13 “Propagation by diffraction”

The (First) Fresnel ellipsoid (1/2)

- Definition: $|TxP_k| + |RxP_k| = R + \frac{\lambda}{2}$

- Fresnel radius: $\rho_1 = \sqrt{\lambda \cdot \frac{a \cdot b}{a + b}}$

$$\rho_{1,\text{max}} = \frac{\sqrt{\lambda \cdot R}}{2}$$

- In LoS condition, objects placed outside the Fresnel ellipsoid (also known as Fresnel zone) may generate new rays, but they have only small effect on the direct ray field. Objects outside the Fresnel ellipsoid are often neglected, since they generate negligible effects.

The (First) Fresnel ellipsoid (2/2)

- Example: knife-edge inside the Fresnel ellipsoid

In order that the presence of the KE produces negligible effect, $\nu < -\sqrt{2}$ must be required (additional loss lower than 1 dB)

$$\nu < -\sqrt{2} \Rightarrow h < -\sqrt{\frac{\lambda \cdot a \cdot b}{a + b}} = -\rho_1 \Rightarrow |h| > \rho_1$$

**The KE must be outside the Fresnel ellipsoid!!**
Fresnel zone clearance

- In the design of point-to-point wireless link, the clearance of the first Fresnel zone is pursued through a proper choice of the communication frequency and of the positioning of both the transmitter and the receiver;

- In case no objects are present inside the Fresnel zone, propagation approximately occurs as in free space;

KE and Diffracted Rays (1/3)

- Assuming a plane wave illumination in the KE problem, the FK integral can be solved and the field beyond the KE can be computed in closed-form[*]

\[
E_{Rx} = A \frac{j\beta}{4\pi} \int_0^\infty \int_0^\infty (1 + \cos \chi) \cdot \frac{e^{-j\beta \rho}}{\rho} \, dy \, dx
\]

Stationary phase method

\[
E_{Rx} = A_0 \cdot e^{-j\beta x} \cdot U(\theta) + A_0 \cdot e^{-j\beta x/\sqrt{\rho}} \cdot \frac{e^{-j\beta \rho}}{\sqrt{\rho}} \cdot D(\theta)
\]

A diffracted wave is generated by the presence of the knife-edge and under the hypothesis made it is a cylindrical wave.

The wave surfaces are therefore cylinders having the axis coincident with the edge of the knife-edge thus we can define the **Diffracted Rays** propagating from the obstacle edge in the radial direction.

\[
D(\theta) = \frac{-\frac{1}{\sqrt{2\pi\beta}}} {2\sin\theta} \cdot \frac{1+\cos\theta}{2} : \text{Diffraction coefficient}
\]

While a closed-form expression for single knife edge diffraction is available, it is not so for multiple knife edge diffraction. There are solutions for two knife edges (Millington) but only iterative solutions are available for a higher number of k.e.’s.

Therefore **heuristic** methods have been developed which consist of arbitrary geometric constructions conceived so as to resort to multiple computations of single-knife-edge diffraction losses.
The rubber-band/Epstein-Peterson method

- The **rubber-band method** is a profile-simplification method:
  - An ideal elastic band is stretched over the link profile. Only those knife-edges that are touched by the band are selected, the others are dropped

- The E.P method is based on a decomposition of the path in sub-paths each one experiencing only one knife-edge diffraction

- The excess loss is computed as a product of each single sub-path loss. The E.P method is not very accurate for a number of obstacles > 4-5

The Epstein-Peterson method (II)

A partial path is associated to each obstacle which spans from the preceding to the following obstacles (virtual Tx and Rx, respectively)

\[
\sqrt{L_{S_{TOT}}} = \prod_{i=1}^{n_o} \left| \frac{1}{2} \int_{v_{0i}}^{\infty} e^{-\frac{\nu^2}{2}} d\nu \right|
\]

\[v_{0i}\] is the Fresnel’s parameter for the i-th obstacle

\[v_{0i} = h_i \sqrt{\frac{2 a_i + b_i}{\lambda a_i b_i}}\]

\[h_i, a_i, b_i\] are shown in the figure. Notice that the \(h_i\) values are measured from the Tx-Rx line of sight.
The Saunders and Bonar method
(outline)

It’s the combination of two different methods

**Flat edge method:** computes the loss due to a uniform series of knife-edges

**Voegler’s method**[*]: allows the computation of the loss due to an arbitrary series of knife-edges (limited number of k.e.’s for CPU time reasons) using a recursive algorithm

At first the loss $L_1$ due to a mean, uniformized profile is computed with the Flat Edge method. Then the original profile is simplified (e.g. with the tight rope method) reducing it to a low number of knife edges. Thus the loss $L_2$ is computed and the final excess loss is obtained as a combination of $L_1$ and $L_2$

The Saunders and Bonar method, although quite complex is the most accurate heuristic multi knife-edge method.


Additional notes on ORT methods

* Multi knife-edge methods are often used for Over-Roof-Top (ORT) propagation prediction

*ORT models only predict multiple diffraction loss along the radial

* **Roof-to-street** propagation, including reflections etc. is not included

* Specific ray models for roof-to-street propagation have been developed
Interaction with real objects

- Reflection/Refraction/Diffraction modeling traditionally refers to flat, smooth, homogeneous half-space boundary or layer with infinite extension. These assumptions are often unsuited to represent real obstacles.

- Nevertheless, the results achieved for the ideal representation still hold for real objects provided that:
  - curvature radius ($R_C$) much larger than $\lambda$;
  - ‘roughness degree’ much smaller than $\lambda$;
  - Rayleigh criterion: $\sigma_h < \lambda/(8\cos\theta_i)$
  - linear extension much larger than $\lambda$;

Otherwise, the object behaves as diffusive scatterer, i.e. it spreads the impinging energy in all directions.

Scatterer / Reflector

- Real objects are quite often neither ideal reflector nor ideal scatterer: the impinging power on a surface element $dS$ is partly reflected in the specular direction, partly scattered in different directions.

- A scattering lobe steered in the direction of specular reflection can be often hypothesized;
Radar Cross Section

- The overall scattering properties (including reflection and diffraction) of an object can be accounted by its Radar Cross Section (RCS):

\[
\sigma(\theta, \phi) = 4\pi r^2 \frac{S_{\text{scat}}(r, \theta, \phi)}{S_{\text{inc}}} = 4\pi r^2 \left| \frac{E_{\text{scat}}(r, \theta, \phi)}{E_{\text{inc}}} \right|^2 \quad [m^2]
\]

\[
\sigma_T = \frac{1}{4\pi} \int_0^{2\pi} \sigma(\theta_s, \phi_s) \, d\Omega = \frac{P_{\text{scat, tot}}}{S_{\text{inc}}}
\]

- Example: in the framework of military application, stealth technology aims at designing aircrafts with an RCS as low as possible, in order to make them undetectable by radar (Sukhoi T-50 seems to have the same RCS of a tennis ball)

- According to its definition, RCS is often computed through the direct measurement of the scattered field. Only in few, ideal cases, expressions of the RCS as a function of the object geometrical and electromagnetic properties are available.

RCS of a metallic plate

- Example: rectangular metallic flat, smooth plate of size a, b:

\[
\sigma(\theta, \phi) = 4\pi \left( \frac{ab}{\lambda} \right)^2 \left( \cos^2 \theta \cdot \cos^2 \phi + \sin^2 \phi \right) \cdot \sin^2 \left( \frac{X}{\pi} \right) \cdot \sin^2 \left( \frac{Y}{\pi} \right)
\]

\[
X = 2 \cdot \sin \theta \cdot \cos \phi
\]

\[
Y = 2 \cdot \sin \theta \cdot \sin \phi
\]
The ‘Effective Roughness’ model

• It's a scalar, incoherent model aiming at simply but effectively describe diffuse scattering (DS) from a surface element \( dS \);

• the power density \( p \) of the scattered field is proportional to a *scattering coefficient* \( S \), related to the overall amount of diffuse power, and is modeled according to a proper *scattering pattern*, describing its spatial distribution;

• DS occurs at the expenses of specular reflection and transmission, and therefore a proper reduction factor \( 0 < R < 1 \) must be applied to properly reduce reflection/transmission coefficients, in order to satisfy the overall power balance.

\[
\begin{align*}
\phi(\psi) & \propto S^2 \left( \frac{1 + \cos \psi}{2} \right)^\alpha_R \\
\phi(\psi) & \propto S^2 \left( \frac{1 + \cos \psi}{2} \right)^\alpha_T
\end{align*}
\]

- \( \psi_R/\psi_T \): angle between the reflection/transmission direction and the scattering direction
- \( \psi_R/\psi_T \) sets the width of the scattering lobe the backward/forward half space

• Several studies have proved that a single-lobe pattern centered on the direction of the specular reflection/transmission is often the best representation;

• At UHF frequencies, \( S \) ranges from 0.2 to 0.4 in rural environment, while values up to 0.6, have been estimated in more complex scenarios; Typical values of \( \alpha \) range between 2 and 4

Further reading: